Higgs scalar potential in asymptotically safe quantum gravity

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Plan

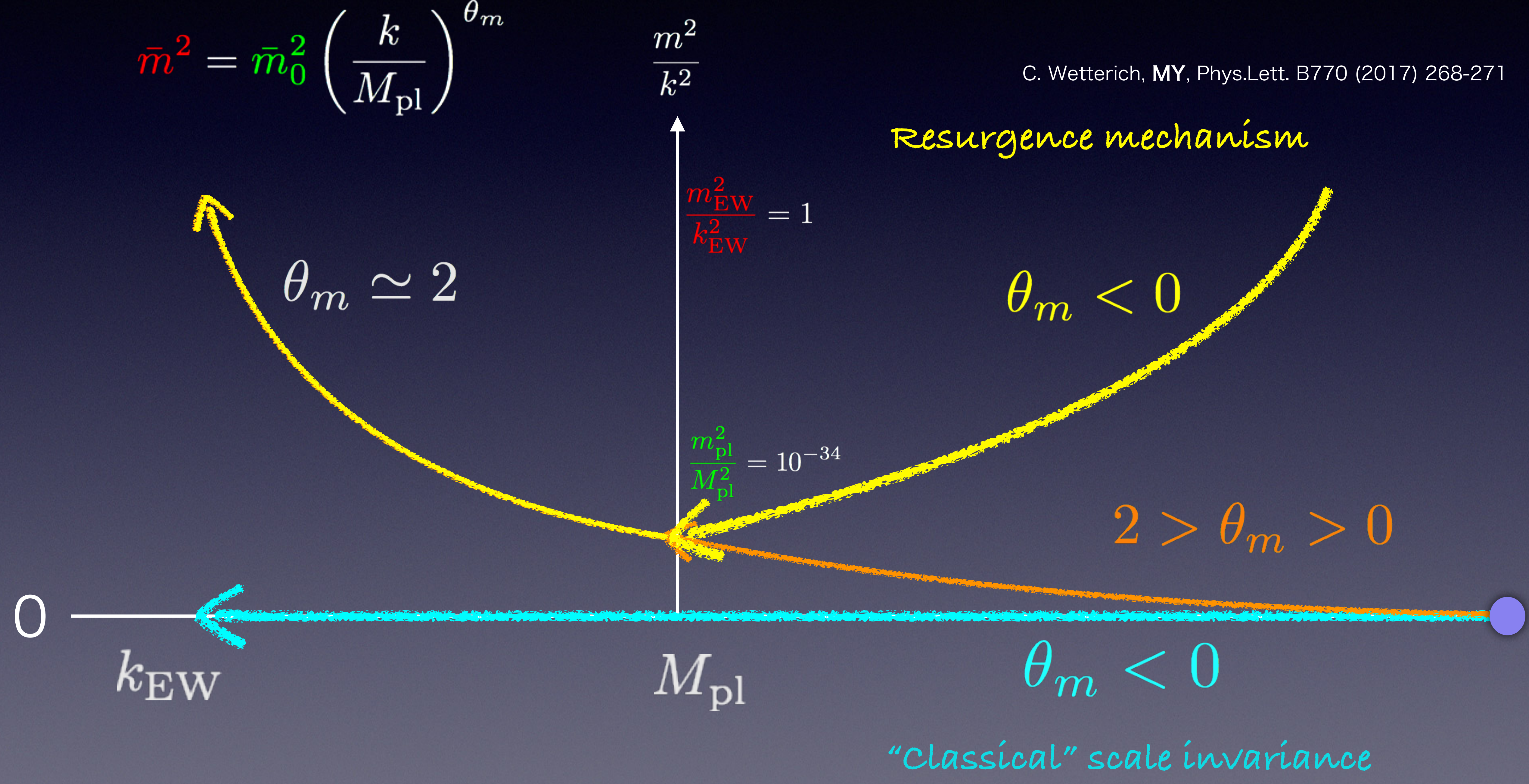
- Quantum scaling
- Gauge invariant flow equation
- Quantum gravity effects on Higgs scalar potential

Quantum Scaling

- Due to graviton fluctuation, a large anomalous dimension could be induced above the Planck scale.
- Energy Scalings deviate from canonical ones.
- Critical exponent (effective dimension) of a scalar mass term

$$\theta_m = 2 - \gamma_{\rm SM} - \frac{5g^*}{3\pi\left(1 - v^*\right)^2} \qquad (0)$$

RG flow of scalar mass



Plan

- Quantum scaling
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Gauge invariant flow equation

- The gauge invariant flow equation

$$\partial_t \Gamma_k = \pi_k - \delta_k \qquad (1)$$

  - π_k : Physical fluctuations $\quad \pi_k = \frac{1}{2}\mathrm{Str}\left\{\partial_t R_P G_P\right\}$

$$\left(\bar{\Gamma}_k^{(2)} + R_P\right) G_P = P^T$$

  - δ_k : a universal measure contribution

- See "C. Wetterich Nucl.Phys. B931 (2018) 262-282" for his derivations.

In background formalism

- The metric $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$
- The metric fluctuation

Clear relations with the Bardeen potentials.

C.Wetterich, Phys.Rev. D95 (2017) no.12, 123525

$$h_{\mu\nu} = Ph_{\mu\nu} + (1-P)h_{\mu\nu}$$
$$= f_{\mu\nu} + a_{\mu\nu}$$

Physical metric fluctuation
$q^{\mu} f_{\mu\nu} = 0$

Gauge metric fluctuation

$$f_{\mu\nu} = t_{\mu\nu} + \frac{1}{3}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\sigma$$

$$a_{\mu\nu} = i\left(q_\mu \kappa_\nu + q_\nu \kappa_\mu\right) + \frac{q_\mu q_\nu}{q^2} u$$

In background formalism

- Einstein-Hilbert truncation + scalar theory

$$\Gamma_k = -\frac{M_{\rm p}^2}{2}\int \mathrm{d}^4x\sqrt{g}R + \int \mathrm{d}^4x\sqrt{g}\left[U(\rho) + \frac{Z_\phi}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right] + S_{\rm gf} + S_{\rm gh} \qquad (2)$$

$$S_{\rm gf} = \frac{1}{2\alpha}\int \mathrm{d}^4x\sqrt{\bar{g}}\bar{g}^{\mu\nu}\Sigma_\mu\Sigma_\nu \qquad \Sigma_\mu = \bar{\nabla}^\nu h_{\nu\mu} - \frac{\beta+1}{4}\bar{\nabla}_\mu h$$

$$S_{\rm gh} = -\int \mathrm{d}^4x\sqrt{\bar{g}}\bar{C}_\mu\left[\bar{g}^{\mu\rho}\bar{\nabla}^2 + \frac{1-\beta}{2}\bar{\nabla}^\mu\bar{\nabla}^\rho + \bar{R}^{\mu\rho}\right]C_\rho$$

- Evaluate the Hessians for

$$h_{\mu\nu} = t_{\mu\nu} + \frac{1}{3}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\sigma + i\left(q_\mu\kappa_\nu + q_\nu\kappa_\mu\right) + \frac{q_\mu q_\nu}{q^2}u$$

$$C_\mu = C_\mu^\perp + iq_\mu C$$

$$\bar{C}_\mu = \bar{C}_\mu^\perp + iq_\mu\bar{C}$$

Hessian for spin 0 modes

$$\Gamma^{(2)}_{(00)} = \begin{array}{c} \\ \sigma \\ u \\ \phi \end{array} \begin{array}{c} \begin{array}{ccc} \sigma & u & \phi \end{array} \\ \begin{pmatrix} \multicolumn{2}{c}{\left(\Gamma^{(2)}_{(00)}\right)_{\text{grav}}} & \frac{U'\phi}{2} \\ & & \frac{U'\phi}{2} \\ \frac{U'\phi}{2} & \frac{U'\phi}{2} & Z_\phi q^2 + U' + 2\rho U'' \end{pmatrix} \end{array} \quad (3)$$

$$\left(\Gamma^{(2)}_{(00)}\right)_{\text{grav}} = \begin{array}{c} \\ \sigma \\ u \end{array} \begin{array}{c} \begin{array}{cc} \sigma & u \end{array} \\ \begin{pmatrix} -\frac{M_p^2}{6}\left(q^2 - \frac{U}{2M_p^2}\right) + \frac{(\beta+1)^2}{16\alpha} q^2 & \frac{U}{4} + \frac{(\beta+1)(\beta-3)}{16\alpha} q^2 \\ \frac{U}{4} + \frac{(\beta+1)(\beta-3)}{16\alpha} q^2 & -\frac{U}{4} + \frac{(\beta-3)^2}{16\alpha} q^2 \end{pmatrix} \end{array} \quad (4)$$

$$\frac{1}{2}\mathrm{Tr}\frac{\partial_t \mathcal{R}_k}{\Gamma^{(2)}_{(00)} + \mathcal{R}_k} \xrightarrow[\alpha \to 0]{\beta = -1} \frac{1}{2}\mathrm{Tr}\frac{\partial_t \mathcal{R}_k}{\Gamma^{(2)}_{(\text{ph})} + \mathcal{R}_k} + \frac{1}{2}\mathrm{Tr}\frac{\partial_t \mathcal{R}_k}{\Gamma^{(2)}_{(\text{gauge})} + \mathcal{R}_k}$$

$$\left(\Gamma^{(2)}_{(00)}\right)_{\text{ph}} = \begin{array}{c} \\ \sigma \\ \phi \end{array} \begin{array}{c} \begin{array}{cc} \sigma & \phi \end{array} \\ \begin{pmatrix} -\frac{M_p^2}{6}\left(q^2 - \frac{U}{2M_p^2}\right) & U'\phi/2 \\ U'\phi/2 & Z_\phi q^2 + U' + 2\rho U'' \end{pmatrix} \end{array} \quad (5)$$

Each contribution

- Hessians

RG flow

Spin 2 (graviton)

$$\left(\Gamma^{(2)}_{(tt)}\right)^{\mu\nu\rho\sigma} = \frac{M_\mathrm{p}^2}{4}\left[q^2 - \frac{2U}{M_\mathrm{p}^2}\right]P^{(t)\mu\nu\rho\sigma}$$

$$\pi_2 = \frac{1}{2}\mathrm{Tr}_{(2)}\left.\frac{\partial_t \mathcal{R}_k}{\Gamma_k^{(2)}+\mathcal{R}_k}\right|_{tt} = \frac{5}{2}\int_q \frac{\partial_t(M_\mathrm{p}^2 R_k)}{M_\mathrm{p}^2(P_k - k^2 v)}$$

Physical contributions

Spin 0

$$\left(\Gamma^{(2)}_{(00)}\right)_\mathrm{ph} = \begin{pmatrix} -\frac{M_\mathrm{p}^2}{6}\left(q^2 - \frac{U}{2M_\mathrm{p}^2}\right) & U'\phi/2 \\ U'\phi/2 & Z_\phi q^2 + U' + 2\rho U'' \end{pmatrix}$$

$$\pi_0 = \frac{1}{2}\int_q \tilde{\partial}_t \ln\left\{3\rho U'^2 + \left(M_\mathrm{p}^2 P_k - \frac{U}{2}\right)(Z_\phi P_k + U' + 2\rho U'')\right\}$$

Spin 1

$$\left(\Gamma^{(2)}_{(\kappa\kappa)}\right)^{\mu\nu} = q^4 P^{(v)\mu\nu} \qquad J_\mathrm{spin\ 1}$$

$$\left(\Gamma^{(2)}_{(\bar{C}^\perp C^\perp)}\right)^{\mu\nu} = q^2 P^{(v)\mu\nu}$$

$$\delta_k^{(1)} = \frac{1}{2}\mathrm{Tr}_{(1)}\left.\frac{\partial_t\mathcal{R}_k}{\Gamma_k^{(2)}+\mathcal{R}_k}\right|_{\kappa\kappa} + J_\mathrm{spin\ 1} = \frac{1}{2}\mathrm{Tr}_{(1)}\frac{\partial_t P_k}{P_k}$$

$$-\epsilon_k^{(1)} = -\mathrm{Tr}_{(1)}\left.\frac{\partial_t\mathcal{R}_k}{\Gamma_k^{(2)}+\mathcal{R}_k}\right|_{\bar{C}^\perp C^\perp} = -\mathrm{Tr}_{(1)}\frac{\partial_t P_k}{P_k}$$

$$\epsilon_k^{(1)} = 2\delta_k^{(1)}$$

Gauge and ghost contributions

Spin 0

$$\Gamma^{(2)}_{(uu)} = q^2$$

$$\Gamma^{(2)}_{(\bar{C}C)} = q^4 \qquad J_\mathrm{spin\ 0}$$

$$\delta_k^{(0)} = \frac{1}{2}\mathrm{Tr}_{(0)}\left.\frac{\partial_t\mathcal{R}_k}{\Gamma_k^{(2)}+\mathcal{R}_k}\right|_{(uu)} = \frac{1}{2}\mathrm{Tr}_{(0)}\frac{\partial_t P_k}{P_k}$$

$$\epsilon_k^{(0)} = 2\delta_k^{(0)}$$

$$-\epsilon_k^{(0)} = -\mathrm{Tr}_{(0)}\left.\frac{\partial_t\mathcal{R}_k}{\Gamma_k^{(2)}+\mathcal{R}_k}\right|_{\bar{C}C} + J_\mathrm{spin\ 0} = -\mathrm{Tr}_{(0)}\frac{\partial_t P_k}{P_k}$$

RG flow

- To summarise…

$$\partial_t \Gamma_k = \pi_2 + \pi_0 + (\delta_k^{(1)} - \epsilon_k^{(1)}) + (\delta_k^{(0)} - \epsilon_k^{(0)})$$
$$= \pi_2 + \pi_0 - \delta_k^{(1)} - \delta_k^{(0)}$$

$$\epsilon_k^{(1)} = 2\delta_k^{(1)}$$
$$\epsilon_k^{(0)} = 2\delta_k^{(0)}$$

DOF 5 + 2 − 3 − 1 = 2 + 1

TT mode (→ 5)

σ and ϕ modes (→ 2)

Two graviton helicities＋scalar (ϕ)

Note

- York decomposition

$$h_{\mu\nu} = t_{\mu\nu} + i(q_\mu \kappa_\nu + q_\nu \kappa_\mu) - \left(q_\mu q_\nu - \frac{1}{4}\eta_{\mu\nu} q^2 \right) s + \frac{1}{4}\eta_{\mu\nu} h$$

- The gauge choice $\beta = 0, \ \alpha \to 0$

$$\partial_t \Gamma_k = \pi_2 + \pi_0 - \delta_k^{(1)} - \delta_k^{(0)}$$

TT mode (π_2) — Trace mode (π_0) $h = \delta^{\mu\nu} h_{\mu\nu}$

- The trace mode does not satisfy the transverse condition.

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RG flow for effective scalar potential

SO(N) scalar and free particles

$$\partial_t \tilde{U} = -4\tilde{U} + (2+\eta_\phi)\tilde{\rho}\tilde{U}'$$

Measure contribution

Spin0 physical mode

graviton

$$+\frac{1}{24\pi^2}\left(1-\frac{\eta_g}{8}\right)\left[\frac{5}{1-v}+\frac{1}{1-v/4}\right]+\frac{\Delta N-4}{32\pi^2}$$

SO(N) scalar modes

$$+\frac{1}{32\pi^2}\left(1-\frac{\eta_\phi}{6}\right)\left[\frac{1}{1+\tilde{U}'+2\tilde{\rho}\tilde{U}''}+\frac{N-1}{1+\tilde{U}'}\right] \qquad (6)$$

$$v(\rho)=\frac{2U(\rho)}{M_{\rm p}^2k^2}=\frac{2\tilde{U}(\rho)}{\tilde{M}_{\rm p}^2}$$

$$\Delta N = \Delta N_S + 2N_V - 2N_F$$

Critical exponent

- The effective potential

$$\tilde{U} = \tilde{V} + \tilde{m}_H^2 \tilde{\rho} + \frac{\tilde{\lambda}_H}{2} \tilde{\rho}^2$$

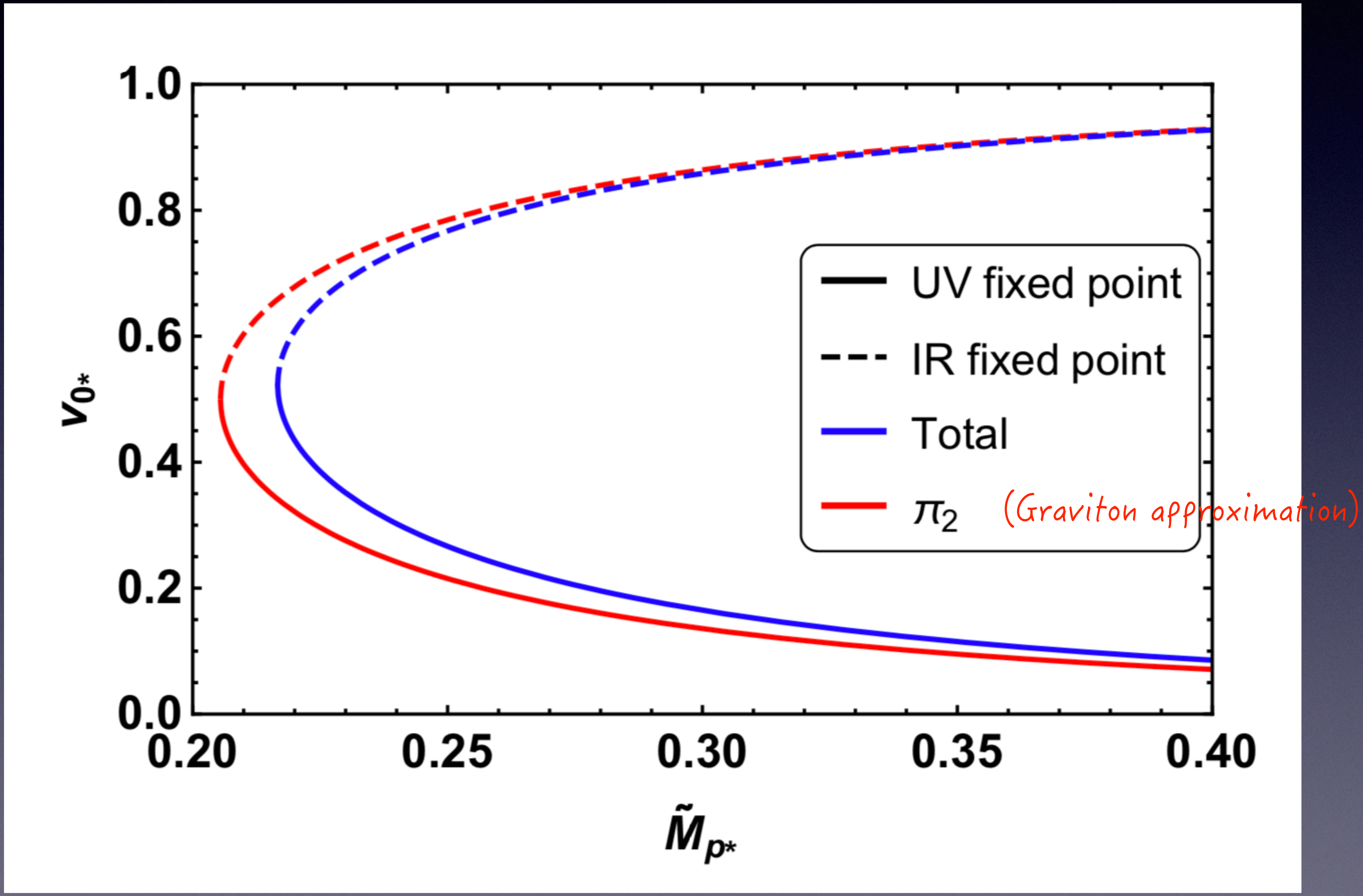
- Stability matrix at a Gaussian-matter FP

$$T = \begin{pmatrix} 4 - A & \frac{1}{4\pi^2 \tilde{M}_{\rm p}^2} & 0 \\ 0 & 2 - A & \frac{3}{16\pi^2} \\ 0 & 0 & -A \end{pmatrix}$$

Anomalous dimension

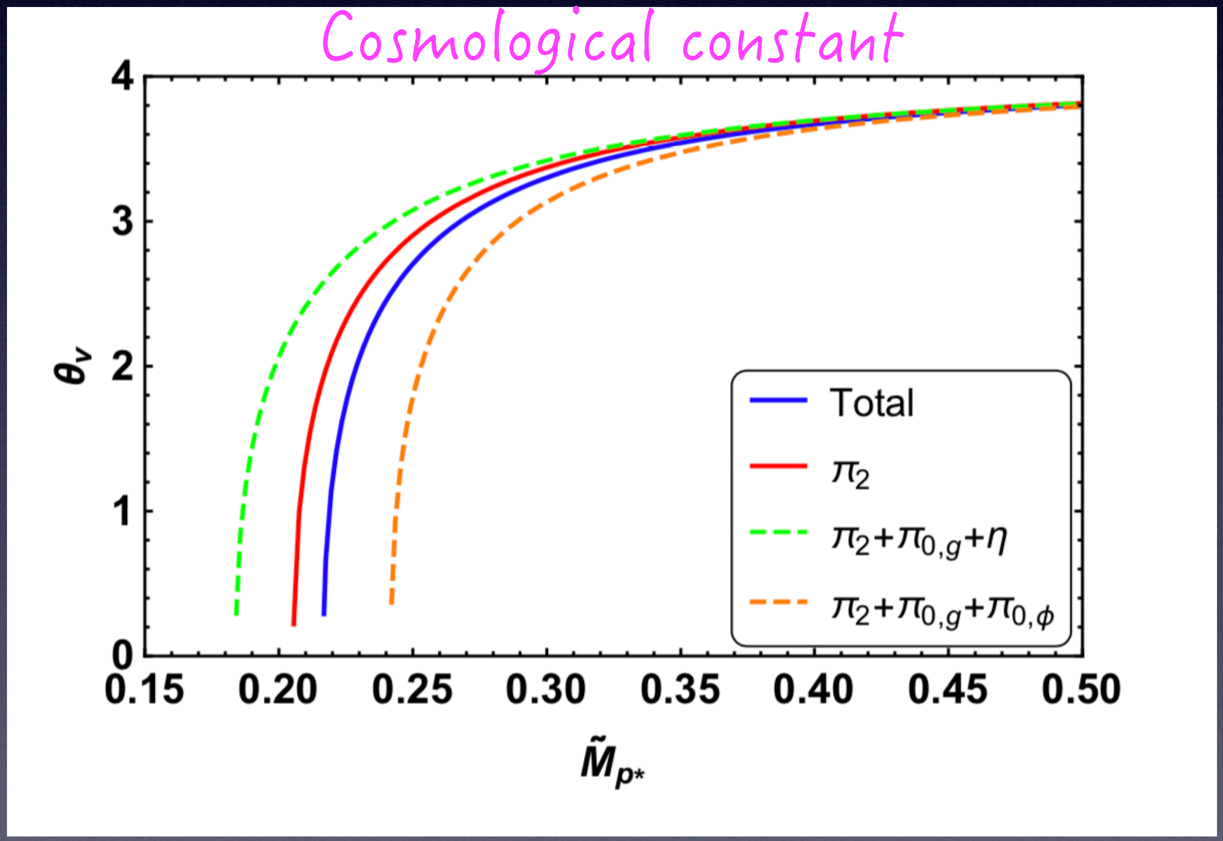
$$A = \frac{1}{48\pi^2 \tilde{M}_{\rm p}^2} \left[\frac{20}{(1 - v_0)^2} + \frac{1}{(1 - v_0/4)^2} \right]$$

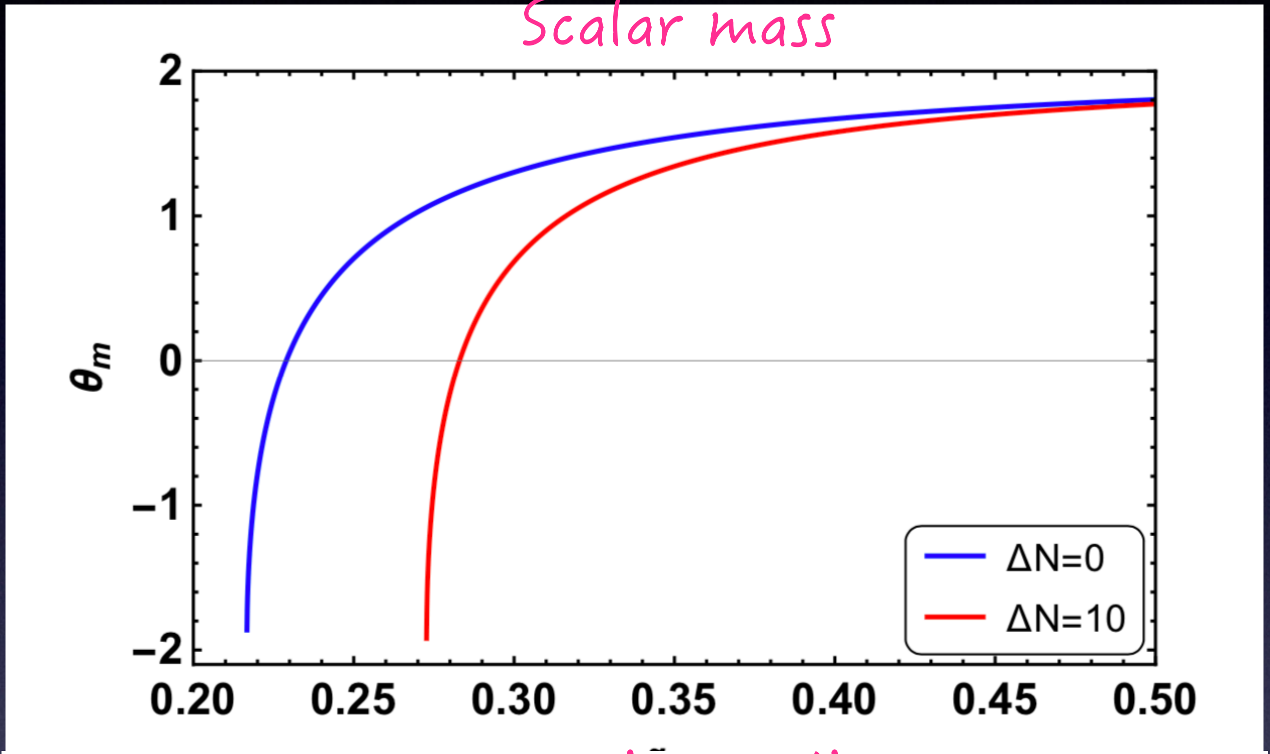
Fixed point of v_{0*}

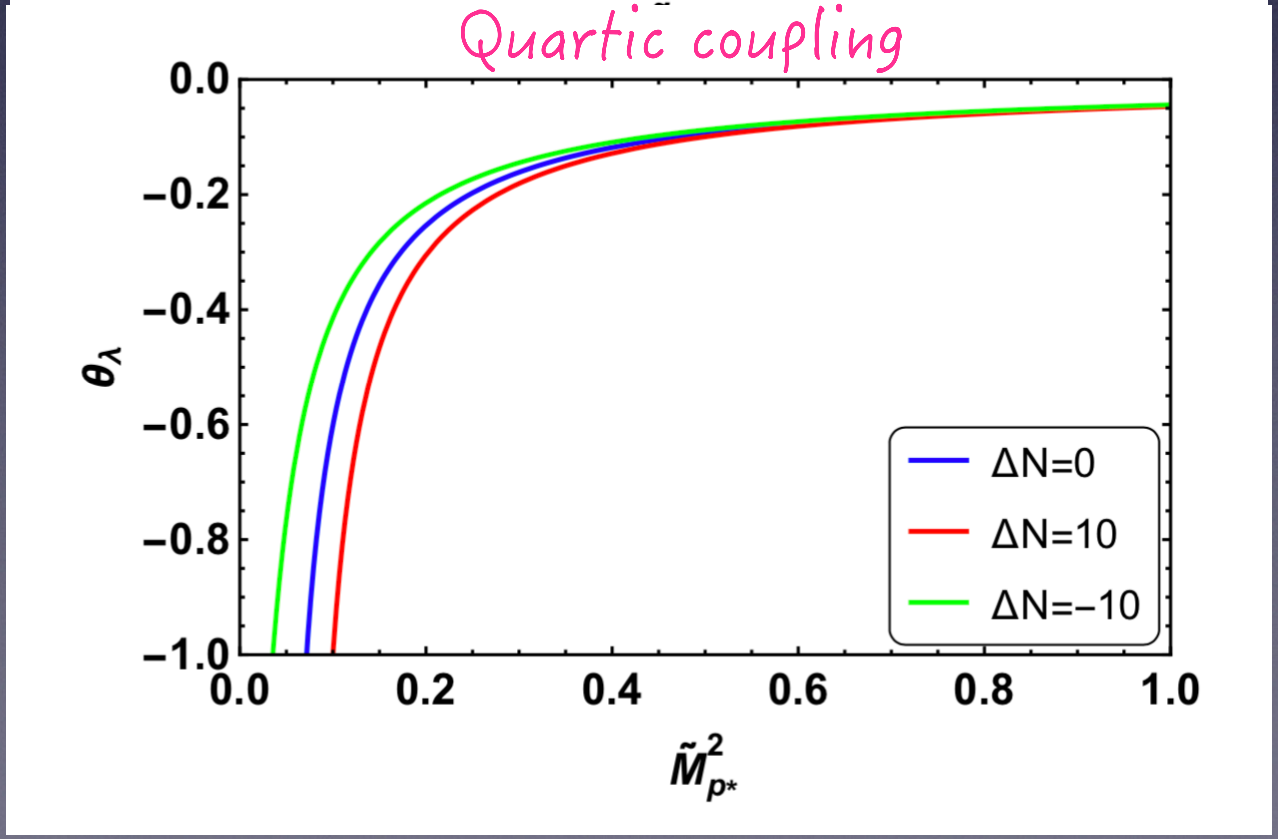


Critical exponents

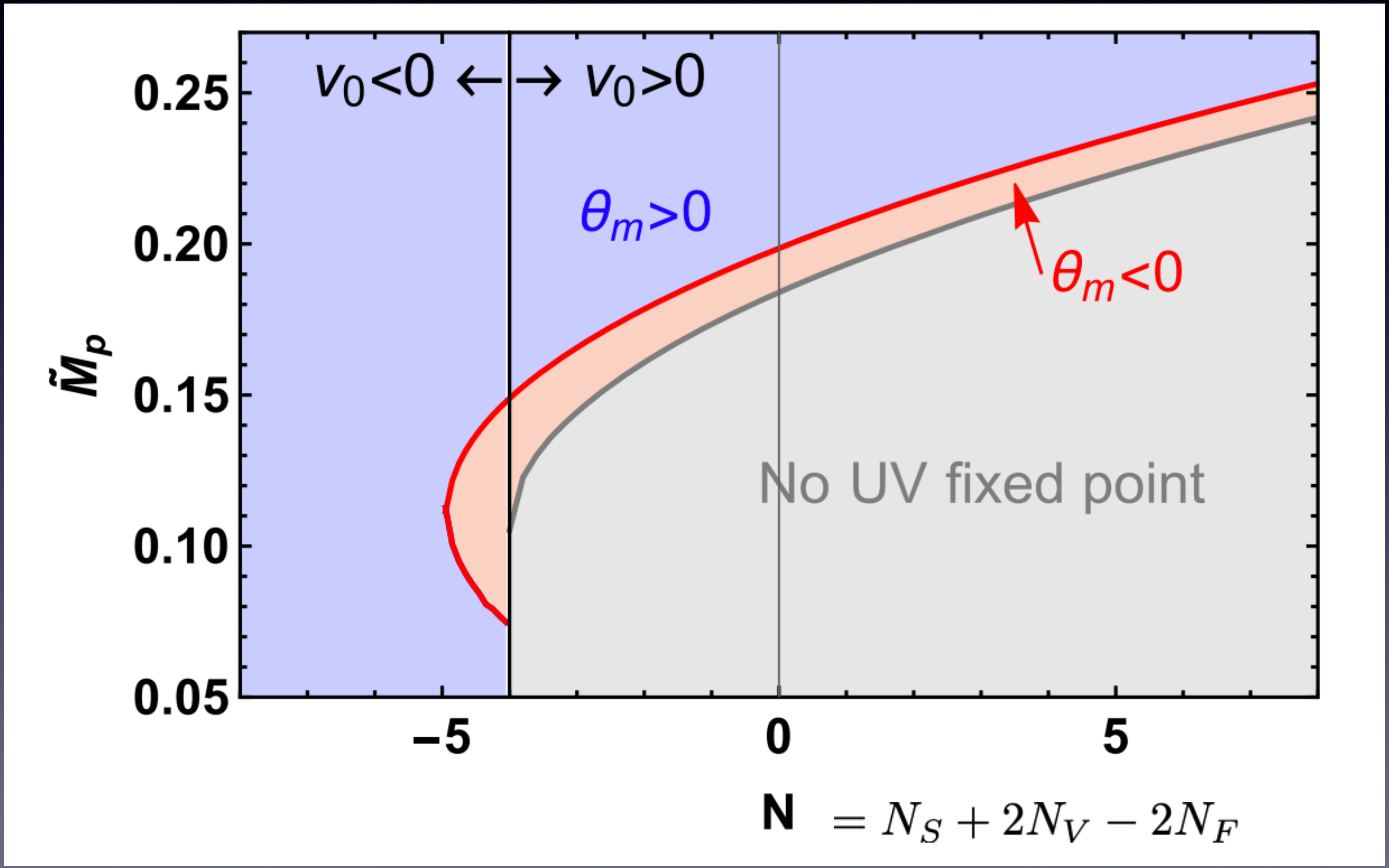
$N = 4,\ \Delta N = 0$







N vs. Mp



v_0<0 ← → v_0>0
θ_m>0
θ_m<0
No UV fixed point
0.25
0.20
0.15
0.10
0.05
$\tilde{M}_p$
–5
0
5
N $= N_S + 2N_V - 2N_F$

Summary

- Asymptotically safe gravity induces large anomalous dimension.
- Scaling of Higgs scalar potential above the Planck scale is changed from canonical scaling.
- Phase diagram on (N, Mp)-plane.