Form Factors in Asymptotic Safety

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L. Bosma, B. Knorr and F.S., arXiv:1904.04845

B. Knorr, C. Ripken and F.S., arXiv:1907.02903

International Asymptotic Safety Seminar

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Outline

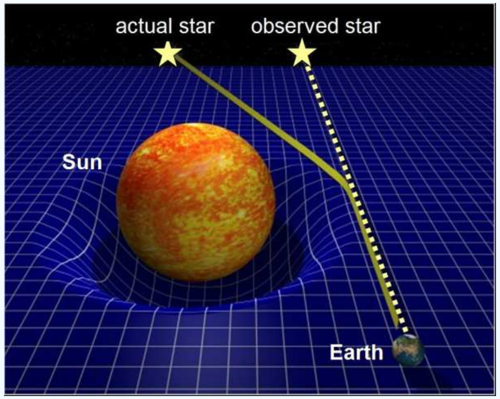
- Motivation
- Form factors – structural aspects
- Form factors – computational remarks
- Example 1: gravitational propagator for transverse-traceless modes
- Example 2: quantum corrected scalar propagator
- Application: quantum corrections to the gravitational potential
- Outlook

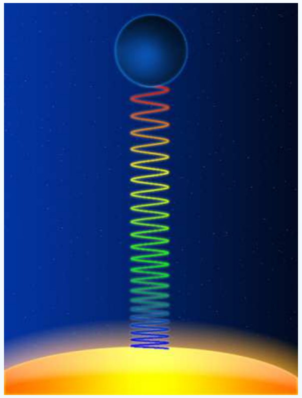
General Relativity

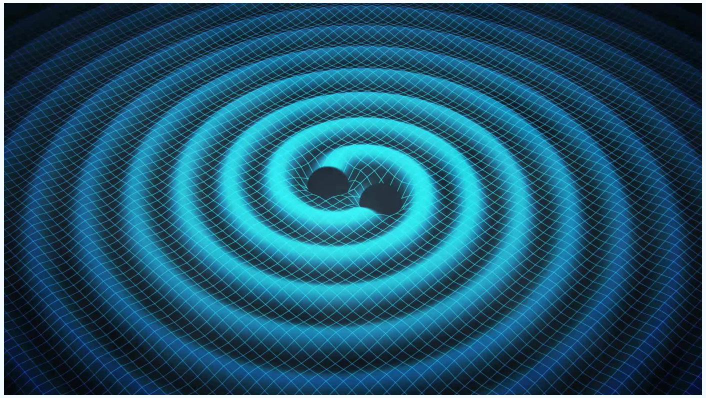
Einstein's equations

$$\underbrace{R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu}\, R + \Lambda\, g_{\mu\nu}}_{\text{dynamics of spacetime}} \quad = \quad \underbrace{8\,\pi\, G_N\, T_{\mu\nu}}_{\text{matter}}$$

experimentally extremely well tested:





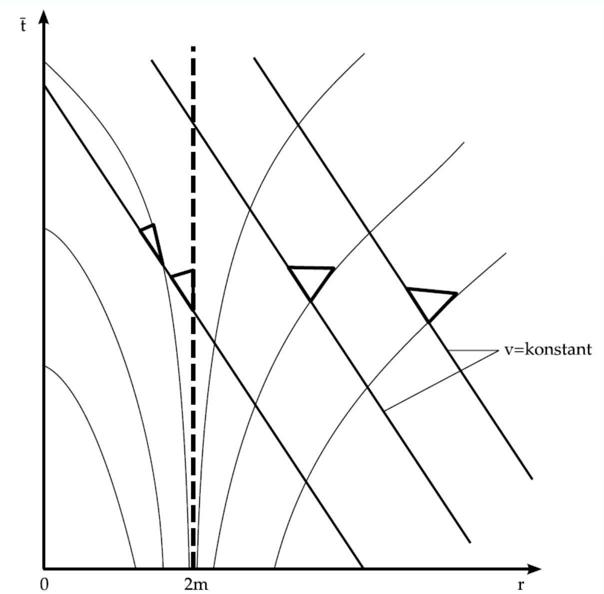


- bending of light rays in the gravitational field
- gravitational redshift
- detection of gravitational waves

Black Holes and Spacetime Singularities

Schwarzschild solution

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$



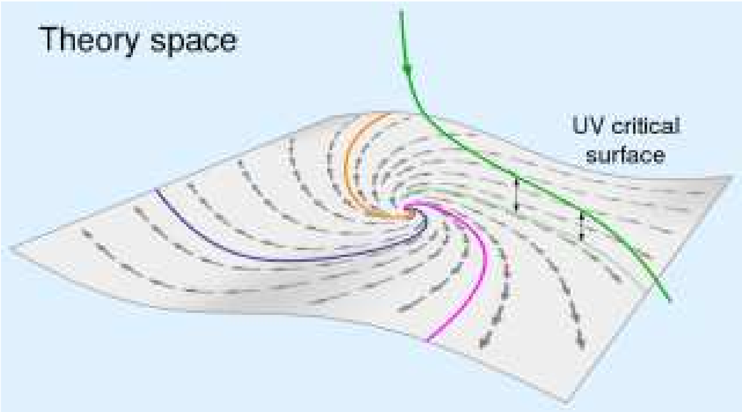
- curvature singularity at $r = 0$

basic objective of quantum gravity

understand the fate of these singularities

Asymptotic Safety Mechanism

high-energy completion controlled by non-Gaussian RG fixed point



[scholarpedia '13]

- 2 classes of RG trajectories:
  - relevant $=$ end at fixed point in UV
  - irrelevant $=$ go somewhere else...
- theory ending at the fixed point is free of unphysical UV divergences
- predictive power:
  - number of relevant directions $\Longleftrightarrow$ free parameters (experimental input)

Wetterich Equation for Gravity

C. Wetterich, Phys. Lett. **B301** (1993) 90
T. Morris, Int. J. Mod. Phys. A9 (1994) 24110
M. Reuter, Phys. Rev. D **57** (1998) 971

central idea: integrate out quantum fluctuations shell-by-shell in momentum-space

implementation: flow equation for effective average action Γ_k:

$$\partial_t \Gamma_k[h_{\mu\nu}; \bar{g}_{\mu\nu}] = \frac{1}{2} \mathrm{Tr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \partial_t \mathcal{R}_k \right]$$

- RG fixed points for theories involving gravity:
  - NGFP for 4-dimensional gravity well-established
  - gravity-matter systems:
    NGFPs with more predictive power than the standard model
- construction uses the background field formalism:
  - depends on two arguments $h_{\mu\nu}, \bar{g}_{\mu\nu}$
  - split symmetry related to $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ restored at $k = 0$

Open Questions

- degrees of freedom associated with the NGFP?
- number of free parameters?
- exact matter-content supporting asymptotic safety?
- low-energy physics compatible with observations?
- restoration of split-symmetry?
- unitarity?

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Investigation method:

- derivative expansion of Γ_k
- derivative expansion insufficient

Derivative expansion of $\Gamma_k^{\mathrm{grav}}[g]$

$\vdots$	$\vdots$		
R^8	$C_{\mu\nu\rho\sigma}\Delta^6 C^{\mu\nu\rho\sigma}$		
R^7	$C_{\mu\nu\rho\sigma}\Delta^5 C^{\mu\nu\rho\sigma}$		
R^6	$C_{\mu\nu\rho\sigma}\Delta^4 C^{\mu\nu\rho\sigma}$		
R^5	$C_{\mu\nu\rho\sigma}\Delta^3 C^{\mu\nu\rho\sigma}$		
R^4	$C_{\mu\nu\rho\sigma}\Delta^2 C^{\mu\nu\rho\sigma}$		
R^3	$C_{\mu\nu\rho\sigma}\Delta C^{\mu\nu\rho\sigma}$	$R\,\Delta R$	+ 5 more
R^2	$C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$	$R_{\mu\nu}R^{\mu\nu}$	
R			
$\mathbb{1}$			

Einstein-Hilbert truncation

Derivative expansion of $\Gamma_k^{\mathrm{grav}}[g]$

Einstein-Hilbert truncation

$C\Box^n C$-truncation

$\vdots$	$\vdots$		
R^8	$C_{\mu\nu\rho\sigma}\Delta^6 C^{\mu\nu\rho\sigma}$		
R^7	$C_{\mu\nu\rho\sigma}\Delta^5 C^{\mu\nu\rho\sigma}$		
R^6	$C_{\mu\nu\rho\sigma}\Delta^4 C^{\mu\nu\rho\sigma}$		
R^5	$C_{\mu\nu\rho\sigma}\Delta^3 C^{\mu\nu\rho\sigma}$		
R^4	$C_{\mu\nu\rho\sigma}\Delta^2 C^{\mu\nu\rho\sigma}$		
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R			
$\mathbb{1}$			

Derivative expansion of $\Gamma_k^{\mathrm{grav}}[g]$

$\vdots$

Einstein-Hilbert truncation

C^2-form factor

R^8

R^7

R^6

R^5

R^4

R^3 $\quad R\,\Delta R \quad$ + 5 more

R^2 $\quad C_{\mu\nu\rho\sigma} W[\Delta] C^{\mu\nu\rho\sigma} \quad R_{\mu\nu} R^{\mu\nu}$

R

$\mathbb{1}$

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derivative expansion $\Longrightarrow$ curvature expansion keeping derivatives

A. Codello and O. Zanusso, Math. Phys. **54** (2013) 013513
S. A. Franchino-Viñas, T. de Paula Netto, I. L. Shapiro and O. Zanusso, Phys. Lett. B **790** (2019) 229

Form Factors

Structural Aspects

Form Factors for Gravity (split-symmetry invariant)

2 form factors at second order in the curvature:

$$\Gamma_k^{\rm C}[g] = \frac{1}{16\pi G_k} \int {\rm d}^d x \sqrt{g}\, C_{\mu\nu\rho\sigma}\, W_k^{\rm C}(\Delta)\, C^{\mu\nu\rho\sigma} \,,$$

$$\Gamma_k^{\rm R}[g] = \frac{1}{16\pi G_k} \int {\rm d}^d x \sqrt{g}\, R\, W_k^{\rm R}(\Delta)\, R \,,$$

enter gravitational propagators in flat space

- working assumption: $W_k(\Delta)$ has representation as Laplace transform

$$W_k(\Delta) = \int_0^\infty ds\, \widetilde{W}_k(s)\, e^{-s\Delta}$$

allows to eliminate a third structure function via

$$\int d^d x \sqrt{g}\, [R^{\rho\sigma\mu\nu} \Delta^n R_{\rho\sigma\mu\nu} - 4 R^{\mu\nu} \Delta^n R_{\mu\nu} + R \Delta^n R] = \mathcal{O}(R^3)\,, n \geq 1$$

- relaxing split symmetry increases the number of form factors

Form Factors: Gravity plus Scalar Matter

I. split-symmetric case:

- 1 form factor for the kinetic term

$$\Gamma_k^{\mathrm{s,kin}}[\phi, g] = \frac{1}{2}\int \mathrm{d}^d x \sqrt{g}\, \phi\, f_k^{(\phi\phi)}(\Delta)\, \phi$$

- 2 form factors at $\mathcal{O}(R)$:

$$\begin{aligned} \Gamma_k^{(\mathrm{R}\phi\phi)}[\phi, g] &= \int \mathrm{d}^d x \sqrt{g} f_k^{(\mathrm{R}\phi\phi)}(\Delta_1, \Delta_2, \Delta_3)\, R\phi\phi\,, \\ \Gamma_k^{(\mathrm{Ric}\phi\phi)}[\phi, g] &= \int \mathrm{d}^d x \sqrt{g} f_k^{(\mathrm{Ric}\phi\phi)}(\Delta_1, \Delta_2, \Delta_3)\, R^{\mu\nu}(D_\mu D_\nu \phi)\phi\,. \end{aligned}$$

remarks:

- Δ_i acts on the ith field
- $f_k^{(\mathrm{R}\phi\phi)}(\Delta_1, \Delta_2, \Delta_3)$: choice of arguments canonical (mixed derivatives eliminated by partial integration)

Form Factors: Gravity plus Scalar Matter

II. flat-space vertex expansion:

- 1 form factor for the kinetic term

$$\Gamma_k^{\mathrm{s,kin}}[\phi, g] = \frac{1}{2}\int \mathrm{d}^d x\, \phi\, f_k^{(\phi\phi)}(\Box)\, \phi$$

- 4 form factors at $\mathcal{O}(h)$:

$$\Gamma_k|_{h\phi\phi} = \int \mathrm{d}^d x \left[f_{(\bar{g})}^{(h\phi\phi)}\, \delta^{\mu\nu} + f_{(11)}^{(h\phi\phi)}\, \partial_1^\mu \partial_1^\nu + f_{(22)}^{(h\phi\phi)}\, \partial_2^\mu \partial_2^\nu + f_{(12)}^{(h\phi\phi)}\, \partial_1^\mu \partial_2^\nu \right] h_{\mu\nu} \phi\phi\,.$$

Remarks:

- ∂_i acts on the ith field
- $f_k^{(h\phi\phi)}(\Box_1, \Box_2, \Box_3)$: choice of arguments canonical (mixed derivatives eliminated by partial integration)

relaxing split-symmetry induces additional form factors

Form Factors: Gravity plus Scalar Matter

III. the split-symmetry invariant sector of the flat-space vertex expansion:

- vertex form factors must be expressible through 2 functions:

$$
\begin{aligned}
f^{(h\phi\phi)}_{(\bar{g})} &= \frac{1}{8}\left[f^{(\phi\phi)}(p_2^2) + f^{(\phi\phi)}(p_3^2) - p_1^2 \frac{f^{(\phi\phi)}(p_2^2) - f^{(\phi\phi)}(p_3^2)}{p_2^2 - p_3^2} \right] \\
&\quad + p_1^2 f^{(\mathrm{R}\phi\phi)} - \frac{1}{8}(p_1^2 + p_2^2 - p_3^2)^2 f^{(\mathrm{Ric}\phi\phi)} , \\
f^{(h\phi\phi)}_{(11)} &= f^{(\mathrm{R}\phi\phi)} , \\
f^{(h\phi\phi)}_{(22)} &= \frac{1}{2} \frac{f^{(\phi\phi)}(p_2^2) - f^{(\phi\phi)}(p_3^2)}{p_2^2 - p_3^2} + \frac{1}{2} p_1^2 f^{(\mathrm{Ric}\phi\phi)} , \\
f^{(h\phi\phi)}_{(12)} &= \frac{1}{2} \frac{f^{(\phi\phi)}(p_2^2) - f^{(\phi\phi)}(p_3^2)}{p_2^2 - p_3^2} + \frac{1}{2} (p_1^2 + p_2^2 - p_3^2) f^{(\mathrm{Ric}\phi\phi)} .
\end{aligned}
$$

structural importance of these identities:

- relations may serve as boundary conditions at $k = 0$
- identities of this form may fix an infinite number of parameters

Form Factors

Computational Aspects

Flow Equations for Form Factors

approximation of Γ_k	structure of RG flow	fixed points
finite number of $\mathcal{O}_i$	ODEs	algebraic
field-dependent functions $f(R_1, \cdots, R_n; t)$	PDEs ($n+1$ var.)	PDEs (n var.)
momentum-dependent form factors $f(p_1, \cdots, p_n; t)$	IDEs ($n+1$ var.)	IDEs (n var.)

- ordinary differential equation (ODE)
- partial differential equation (PDE)
- integro-differential equation (IDE)

Extracting n-point vertices retaining all derivatives

example: scalar-kinetic term

$$\Gamma_k^{\mathrm{s,kin}}[\phi, g] = \frac{1}{2} \int \mathrm{d}^d x \sqrt{g}\, \phi\, f_k^{(\phi\phi)}(\Delta)\, \phi$$

- argument of form factor depends on fluctuation field

$$\Delta\, \phi \simeq \Big[\Box + \underbrace{\mathbb{d}_1}_{\mathcal{O}(h)} + \underbrace{\mathbb{d}_2}_{\mathcal{O}(h^2)} + \cdots \Big] \phi$$

strategy:

- express $f_k(\Delta)$ through its Laplace transform
- use multi-commutators to keep exact momentum dependence

$$\frac{d}{d\epsilon} e^{X+\epsilon Y} = \sum_{j=0}^{\infty} \frac{1}{(j+1)!} [X + \epsilon Y, Y]_j \, e^{X+\epsilon Y}$$

where

$$[X, Y]_l \equiv [X, [X, Y]_{l-1}]\,, \qquad [X, Y]_0 = Y\,, \quad l \geq 0 \in \mathbb{N}.$$

Extracting n-point vertices retaining all derivatives

example: scalar-kinetic term

$$\Gamma_k^{\mathrm{s,kin}}[\phi, g] = \frac{1}{2}\int \mathrm{d}^d x \sqrt{g}\, \phi\, f_k^{(\phi\phi)}(\Delta)\, \phi$$

the $h\phi\phi$-vertex

$$\begin{aligned}\Gamma_k^{\mathrm{kin}}|_{h\phi\phi} \quad &= \quad \frac{1}{2}\int d^d x \,\Big[\frac{1}{2}\, f^{(\phi\phi)}(\Box_2) h\phi\phi \\ &+ \int_0^\infty ds\, \tilde{f}^{(\phi\phi)}(s) \sum_{j\geq 0}^{\infty} \frac{(-s)^{j+1}}{(j+1)!} \sum_{l=0}^{j} \binom{j}{l} (-1)^l (\Box^{j-l}\phi)\, \mathrm{d}_1\, \Box^l\, e^{-s\Box}\phi\Big]\end{aligned}$$

notice: sums and integral transforms can be done analytically

$$\begin{aligned}&\int_0^\infty \mathrm{d}s\, \tilde{f}^{(\phi\phi)}(s) \sum_{j\geq 0}^{\infty} \frac{(-s)^{j+1}}{(j+1)!} \sum_{l=0}^{j} \binom{j}{l} (-1)^l \Box_2^{j-l}\, \Box_3^l\, e^{-s\Box_3} h\phi\phi \\ &= \int_0^\infty \mathrm{d}s\, \tilde{f}^{(\phi\phi)}(s) \sum_{j\geq 0}^{\infty} \frac{(-s)^{j+1}}{(j+1)!} (\Box_2 - \Box_3)^j\; e^{-s\Box_3} h\phi\phi \\ &= (\Box_2 - \Box_3)^{-1} \left(f^{(\phi\phi)}(\Box_2) - f^{(\phi\phi)}(\Box_3)\right) h\phi\phi\,.\end{aligned}$$

Computing Form Factors I

The Gravitational Sector

The C^2-Form Factor (Conformally Reduced Setting)

ansatz

$$\Gamma_k^{\text{grav}} = \frac{1}{16\pi G_k} \int d^4x \sqrt{g} \left[2\Lambda_k - R + C_{\mu\nu\rho\sigma} \, W_k(\Delta) \, C^{\mu\nu\rho\sigma} \right]$$

- $W_k(\Delta)$ gives corrections to transverse-traceless propagator

$$\mathcal{G}^{\text{TT}}(q^2) \propto \Big(\underbrace{q^2}_{\text{Einstein}-\text{Hilbert}} + \underbrace{2 \, (q^2)^2 \, W_k(q^2)}_{\text{form factor}} \Big)^{-1}$$

work in conformally reduced setting:

- considers quantum fluctuations in the conformal factor only

$$g_{\mu\nu} = \left(1 + \tfrac{1}{4} h\right) \hat{g}_{\mu\nu}$$

- flow of $w(q^2) \equiv k^{-2} \, W_k(\Delta/k^2)$ from Wetterich equation

Flow Equation for the Form Factor

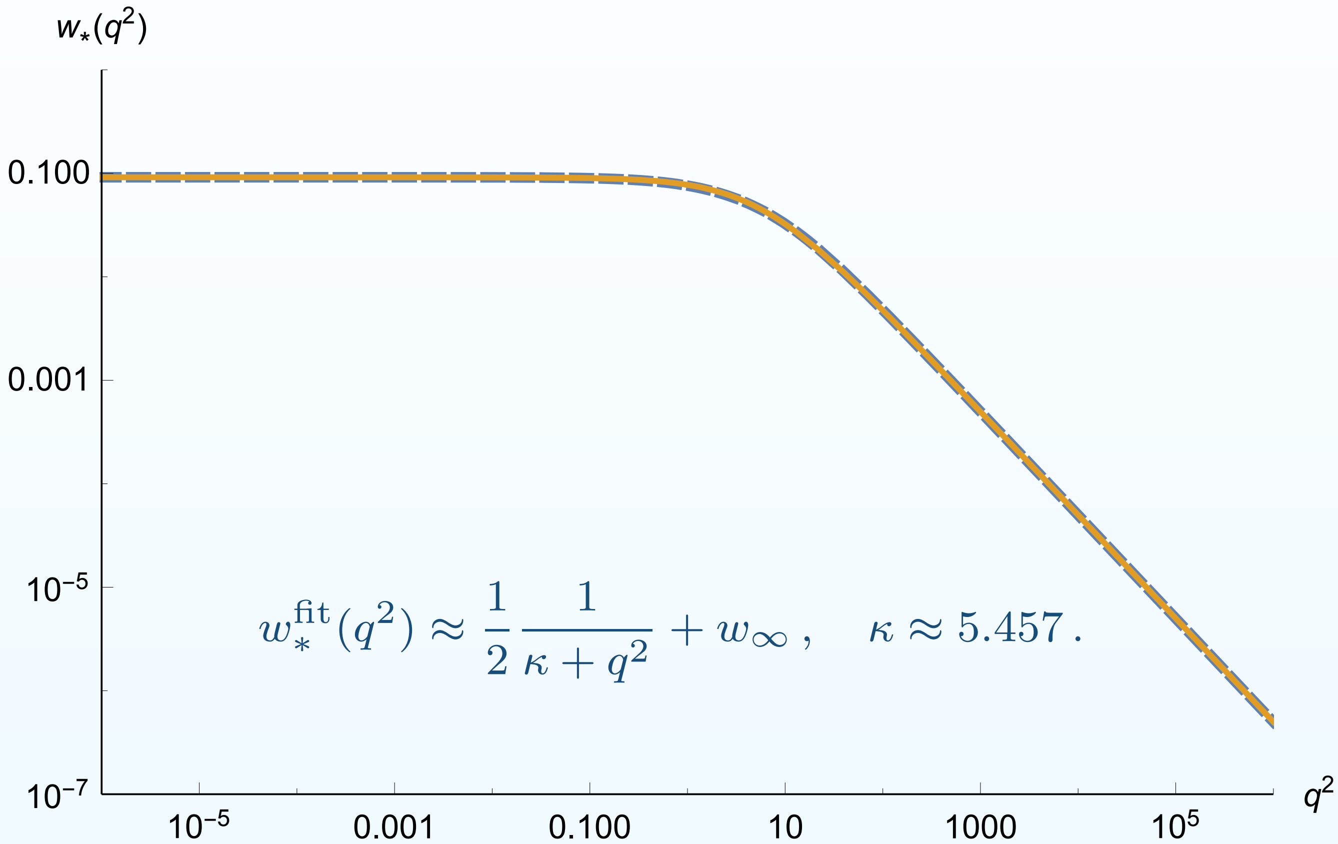
Einstein-Hilbert sector similar to the Einstein-Hilbert truncation

$$
\begin{aligned}
k\partial_k w(q^2) &= (2+\eta_N) w(q^2) + 2q^2 w'(q^2) \\
&\quad + \frac{g}{24\pi}\int_0^{\frac{1}{4}} \mathrm{d}u\, (1-4u)^{\frac{3}{2}} \frac{(2-\eta_N)R(uq^2) - 2uq^2 R'(uq^2)}{uq^2 + R(uq^2) + \mu} \\
&\quad + \frac{16g}{3\pi^2}\int_0^{\infty} \mathrm{d}p \int_{-1}^{1} \mathrm{d}x\, p^3 \sqrt{1-x^2}\, \frac{(2-\eta_N)R(p^2) - 2p^2 R'(p^2)}{(p^2 + R(p^2) + \mu)^2} \\
&\qquad \Big[\tfrac{1}{8} \big(w(p^2 + 2pqx + q^2) - w(q^2) \big) \\
&\qquad + \tfrac{2q^4 + 4(q^2 - p^2)(pqx) + p^2 q^2 (7 - 6x^2)}{16(p^2 + 2pqx)^2} \big(w(p^2 + 2pqx + q^2) - w(q^2) \big) \\
&\qquad + \tfrac{3p^4 - 2q^4 + 22p^2(pqx) - 5p^2 q^2 (1 - 6x^2)}{16(p^2 + 2pqx)} w'(q^2) \Big] .
\end{aligned}
$$

- inhomogeneous term $\Longrightarrow$ form factor is induced
- integro-differential equation requires knowing $w(x)$ on positive real axis

The Form Factor

solving fixed point equation with pseudo-spectral methods:



- w_∞ undetermined constant (lifted in full computation)
- expansion: $w_*^{\rm fit}(q^2)$ is an infinite power series in q^2

  $\Longrightarrow$ avoids Ostrogradski instability

Computing Form Factors II

The Scalar Kinetic Term

The Scalar-Kinetic Form Factor

ansatz

$$\Gamma_k[g, \phi, \bar{c}, c; \bar{g}] \approx \Gamma_k^{\mathrm{grav}}[g] + \Gamma_k^{\mathrm{scalar}}[\phi, g] + \Gamma_k^{\mathrm{gf}}[g; \bar{g}] + S^{\mathrm{gh}}[g, \bar{c}, c; \bar{g}] \,.$$

- gravitational sector:

$$\Gamma_k^{\mathrm{grav}}[g] = \frac{1}{16\pi G_k} \int \mathrm{d}^d x \sqrt{g} \, [2\Lambda_k - R]$$

  - supplemented by harmonic gauge

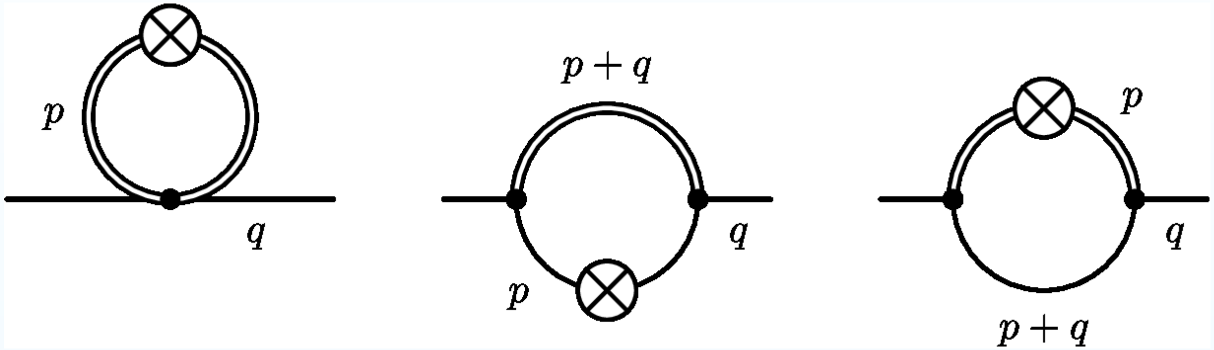
- scalar sector:

$$\Gamma_k^{\mathrm{scalar}}[\phi, g] = \frac{1}{2} Z_k^s \int \mathrm{d}^d x \sqrt{g} \phi \bar{f}_k(\Delta) \phi$$

  - Z_k^s: momentum-independent wave-function renormalization
  - $\bar{f}_k(\Delta)$: dimensionful form factor

Flow Equation for the Scalar Form Factor

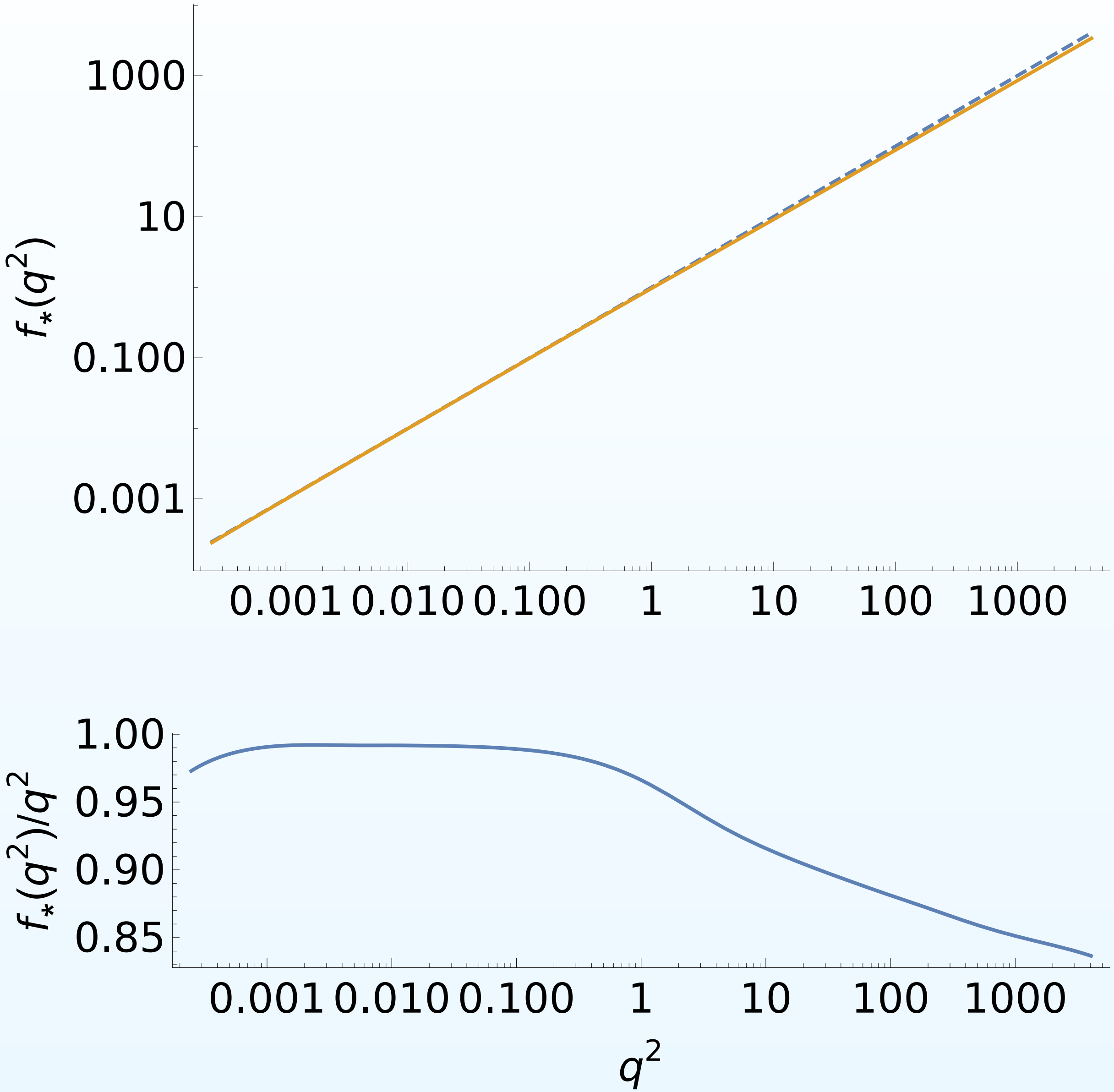
momentum-dependence of the scalar 2-point function



integro-differential equation encoding the fixed point structure of $f_*(p^2)$

- non-linear in $f_*(p^2)$
- inhomogeneous: $f_*(p^2)$ is generated by quantum fluctuations
- non-trivial: $f_*(p^2) \propto p^2$ is not a solution
- form-factor feeds back into the Einstein-Hilbert sector

The Scalar-Kinetic Form Factor at the NGFP



Properties of the Form Factor Solution

stability analysis

- minimally coupled scalar field ($Z_k^{\mathrm{s}} = 1, \bar{f}_k(\Delta) = \Delta$)

$$\theta_{1,2} = 1.64 \pm 4.12\mathrm{i}$$

- including the Form Factor (θ_3 associated with gap parameter)

$$\theta_{1,2} = 1.64 \pm 4.14\mathbf{i}\,, \qquad \theta_3 = 1.16$$

asymptotic scaling for large momentum

$$\lim_{p^2 \to \infty} G_*^{\mathrm{s}}(p^2) \propto \frac{1}{p^{2\alpha}}\,, \qquad \alpha = 0.949$$

- short-distance asymptotics of the scalar two-point correlator:

$$\langle \phi(x)\phi(y) \rangle \simeq \frac{1}{|x-y|^{2\Delta}}\,, \quad \Delta = (2-\alpha) = 1.051$$

compatible with unitarity bound $\Delta \geq \Delta_{\mathrm{min}} = 1$

Properties of the Form Factor Solution

reference	η_*^{s}	Δ
Dona, Eichhorn, Percacci [1311.2898]	-0.361	—
Meibohm, Pawlowski, Reichert [1510.07018]	0	—
Becker, Ripken, Saueressig [1709.09098]	-0.771	—
full form factor	-0.176	1.051

gravity-scalar NGFP is gravity dominated

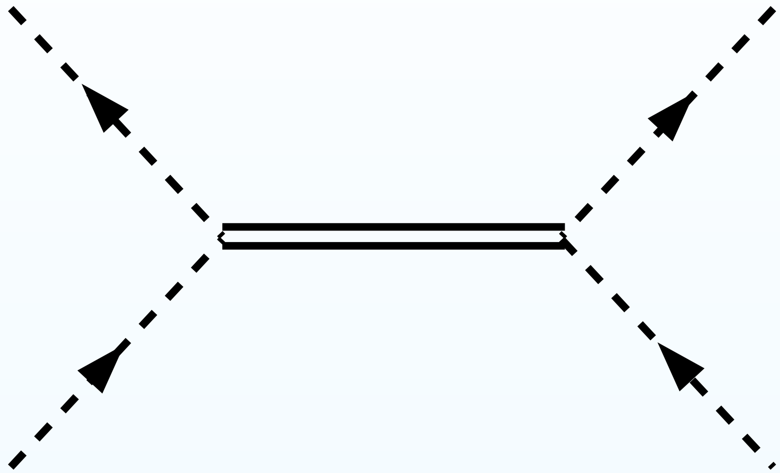
Application

The Newtonian Gravitational Potential

Newtonian Gravitational Potential from Field Theory

Phys. Rev. D50 (1994) 3874

non-relativistic graviton-mediated interaction of two scalar fields: (masses m_1, m_2):



$$V(\mathbf{r}) = -\frac{1}{2m_1}\frac{1}{2m_2}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{r}}\mathcal{M} \; = -\frac{G\, m_1\, m_2}{\mathbf{r}}$$

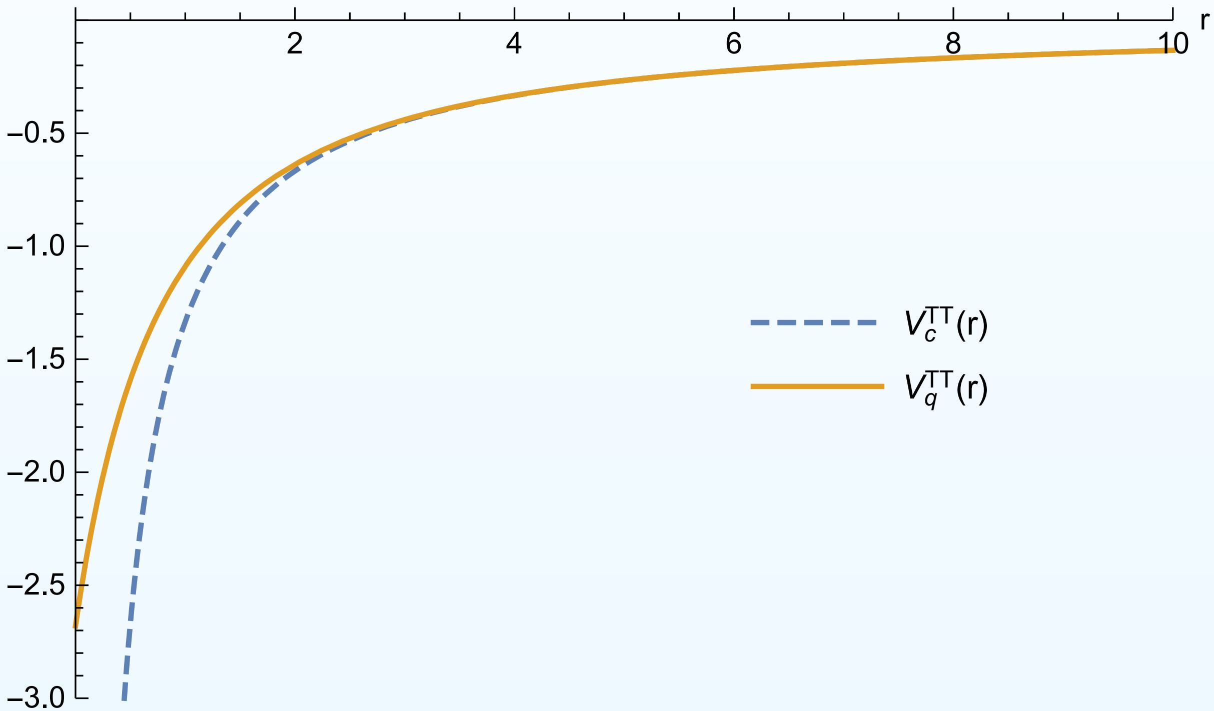
classical scattering amplitude ($q = (0\,,\,\mathbf{q})$)

$$\mathcal{M} = 16\pi G\, m_1^2\, m_2^2\; \mathcal{G}(\mathbf{q}^2) \quad , \qquad \mathcal{G}_{\mathrm{classical}}(\mathbf{q}^2) = \underbrace{\mathbf{q}^{-2}}_{\mathrm{Einstein-Hilbert}}$$

The Quantum-Corrected Newtonian Potential

Strategy:

- restrict scattering amplitude to transverse-traceless contribution
- replace $\mathcal{G}^{\mathrm{TT}}_{\mathrm{classical}}(\mathbf{q}^2) \Longrightarrow \mathcal{G}^{\mathrm{TT}}_{\mathrm{non-perturbative}}(\mathbf{q}^2)$



$V^{\mathrm{TT}}_{\mathrm{quantum}}(\mathbf{r})$ remains finite as $\mathbf{r} \to 0$

Black Hole Singularities - A Wild Speculation

Schwarzschild solution in terms of $V_{\rm c}(\mathbf{r})$:

$$ds^2 = -\left(1 - 2V_{\rm c}(r)\right) dt^2 + \left(1 - V_{\rm c}(r)\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

improve by substituting

$$V_{\rm c}(r) \Longrightarrow V_{\rm q}(r) \simeq \alpha_1 + \alpha_2 r + \ldots$$

modifies the curvature singularity at $r = 0$!

- curvature singularity still present but integrable
- geodesics can pass through $r = 0$

> form factors may be closely related to singularity resolution

Conclusions

Take-away Messages

form factors:

- highly relevant for the dynamics
- we have the technology to compute them self-consistently

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gravity:

- gravitational potential rendered finite

gravity-matter:

- short-distance fall-off of scalar 2-point correlator compatible with unitarity

no propagators contain unitarity-violating poltergeists

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!!! WORK AHEAD !!!