The Fröhlich-Morcchio-Strocchi Mechansim and Quantum Gravity

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Natural Sciences

Der Wissenschaftsfonds.

What is this talk about?

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- Global and local internal symmetries
    - Physical content of symmetries

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    - Two examples in QFT
        - Standard-model Higgs sector
        - Toy grand-unified theory

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  - Two examples in QFT
    - Standard-model Higgs sector
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- Space-time symmetries
- FMS in quantum gravity
- Some speculations on phenomenology

Path integral and global symmetries

$$Z = \int_{\Omega} D\phi^{a} e^{iS[\phi]}$$

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Field – transforms linearly under a group $\phi^a \rightarrow G^{ab}\,\phi^b$

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Measure is invariant
- no anomalies

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Action is invariant

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Integration range
- contains all orbits $G\phi$

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  - There is no absolute orientation/frame in the internal space
  - Does not change when averaging over position
  - There is no absolute charge

Path integral and global symmetries

$$\langle \phi^b(x) \rangle = \int_\Omega D\phi^a e^{iS[\phi]} \phi^b(x) = 0$$

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    - There is no absolute orientation/frame in the internal space
    - Does not change when averaging over position
    - There is no absolute charge
- Individual measurements have a direction
    - These are statements about averages/expectation values

Path integral and global symmetries

$$\langle \phi^b(x)\phi^c(y)\rangle = \int_{\Omega} D\phi^a e^{iS[\phi]}\phi^b(x)\phi^c(y)$$

- Relative charge measurement averaged over all possible starting point

Path integral and global symmetries

$$\langle \phi^b(x)\phi^c(y)\rangle = \int_\Omega D\phi^a e^{iS[\phi]} \phi^b(x)\phi^c(y) = 0$$

- Relative charge measurement averaged over all possible starting point
  - Vanishes because no preferred absolute starting point

Path integral and global symmetries

$$\left\langle \delta_{bc} \phi^b(x) \phi^c(y) \right\rangle$$
$$= \int_\Omega D\phi^a e^{iS[\phi]} \delta_{bc} \phi^b(x) \phi^c(y)$$

- Group-invariant quantity
  - Measures relative orientation
  - Created from an invariant tensor δ_{ab}

Path integral and global symmetries

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$$= \int_\Omega D\phi^a e^{iS[\phi]} \delta_{bc} \phi^b(x) \phi^c(y) \neq 0$$

- Group-invariant quantity
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  - Created from an invariant tensor δ_{ab}
  - Allows to measure degeneracies
    - Generalized Wigner-Eckart theorem

Path integral and global symmetries

$$\langle \phi^b(x) \rangle = \int_\Omega D\phi^a e^{iS[\phi, j^c]} \phi^b(x)$$

- Explicit breaking is an explicit absolute frame

Path integral and global symmetries

External source breaks the symmetry

$$\langle \phi^b(x) \rangle = \int_{\Omega} D\phi^a \, e^{iS[\phi, j^c]} \, \phi^b(x) = v(j^c) \neq 0$$

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  - Remains for $j^c \rightarrow 0$: Spontaneous symmetry breaking
    - Measurement preferably aligned to source direction

Path integral and global symmetries

$$\langle \phi^b(x) \rangle = \int_\Omega D\phi^a e^{iS[\phi,0]} \phi^b(x) = v(0) = 0$$

- Explicit breaking is an explicit absolute frame
  - Results are relative to the source
  - Remains for $j^c \rightarrow 0$: Spontaneous symmetry breaking
    - Measurement preferably aligned to source direction
  - At zero source: Expectation values always vanish
    - Individual measurements can show preferred direction
    - No absolute direction preferred

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$$= \int_\Omega D\phi^a e^{iS[\phi]} \delta_{bc} \phi^b(x) \phi^c(y) = 0$$

- No longer invariant under gauge transformations
  - Vanishes just as any other non-invariant quantity

Path integral and global symmetries

Transporter

$$\langle \phi^b(x) \boxed{U^{bc}(x,y)} \phi^c(y) \rangle$$

$$= \int_\Omega D\phi^a DU\, e^{iS[\phi]} \phi^b(x) U^{bc}(x,y) \phi^c(y)$$

- Transporter compensates gauge transformations
  - Implemented by gauge fields

Path integral and global symmetries

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Path integral and global symmetries

Reduced integration range

$$\langle \phi^b(x)\,\phi^c(y)\rangle$$

$$= \int_{\Omega_c} D\phi^a\, DU\, W(U,\phi)\, e^{iS[\phi]}\, \phi^b(x)\,\phi^c(y) \neq 0$$

- Reduction of integration region by gauge fixing
  - Arbitrary choice of coordinates
  - Weight factor to keep gauge-invariant quantities the same

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- Sources break a symmetry explicitly and create an absolute frame
    - Like an additional term in the theory
- Gauge-fixing is a choice of coordinates in an internal space
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- Only quantities invariant under all symmetries are measurable

Fröhlich-Morchio-Strocchi mechanism

- Is there a gauge, in which gauge-invariant observables are easy to calculate?

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    - Choice different for different parameter sets

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  - Split **after** gauge-fixing fields such that they become classical fields plus quantum corrections

A toy model

- Consider an SU(2) with a single fundamental scalar

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- Essentially the standard model Higgs

$$L=-\frac{1}{4}W_{\mu\nu}^{a}W_{a}^{\mu\nu}$$

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- Coupling g and some numbers f^{abc}

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- Parameters selected for a BEH effect

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- Global SU(2) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only

$$W_{\mu}^{a}\rightarrow W_{\mu}^{a} \qquad h\rightarrow h\Omega$$

Physical states

[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles
  - **Cannot** be the elementary particles
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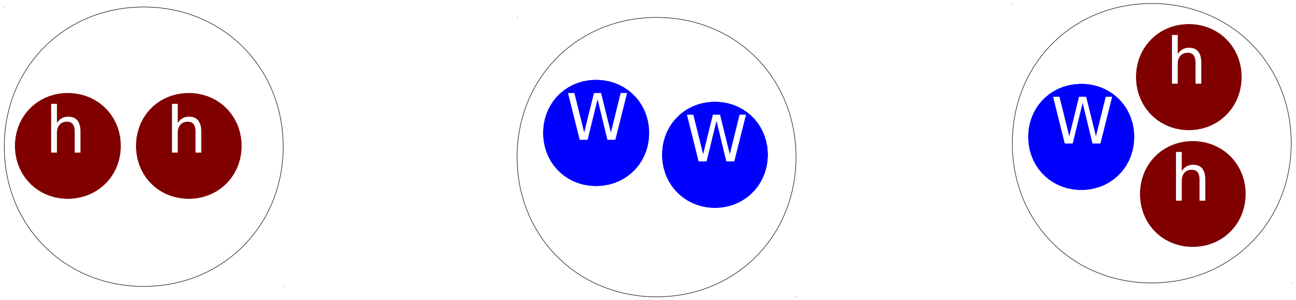
  - Higgs-Higgs, W-W, Higgs-Higgs-W etc.



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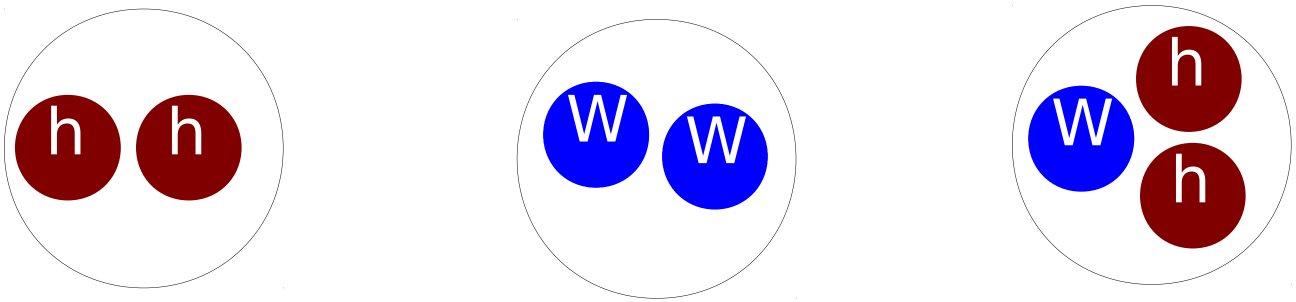

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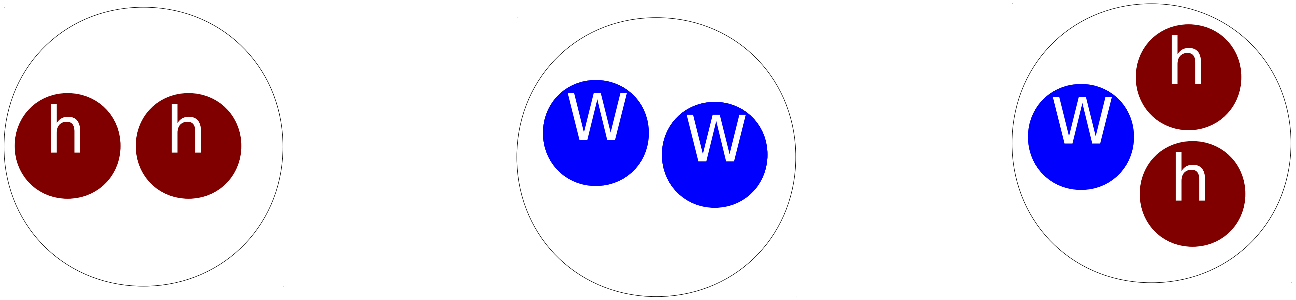

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- Can be described using the FMS mechanism

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- Implementation
  - Choose a gauge which allows for a Higgs vev
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    - Necessary symmetry: translation invariance, Lorentz symmetry
  - Split after gauge-fixing Higgs field in vev and fluctuation field

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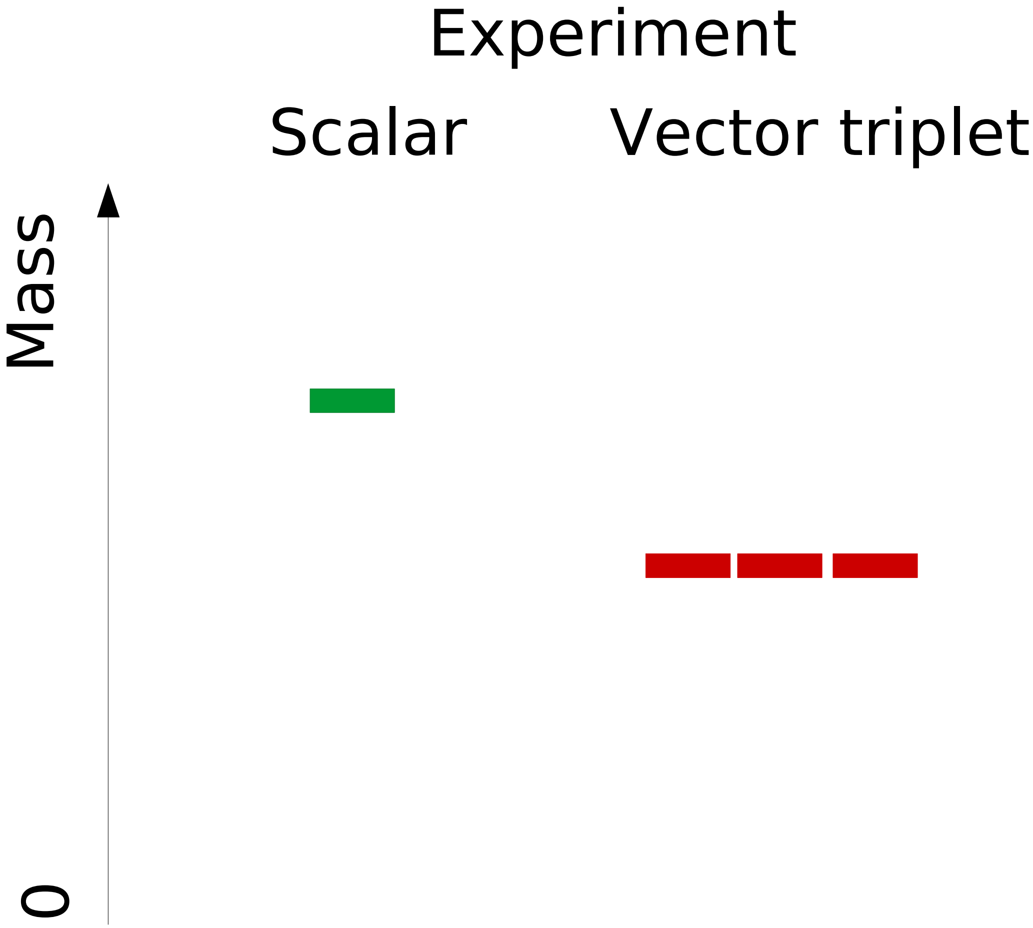
- Test: Calculate gauge-invariant observables

Physical spectrum


Experiment

Scalar

Vector triplet

‘Experiment’: Derived from the situation in the standard model

Do the calculations

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
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- J^{PC} and custodial charge only quantum numbers
    - Different from perturbation theory
        - Operators limited to asymptotic, elementary, gauge-dependent states

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- Formulate gauge-invariant, composite operators
  - Bound state structure – non-perturbative methods! - Lattice

Physical spectrum

[Maas’12, Maas & Mufti’14]

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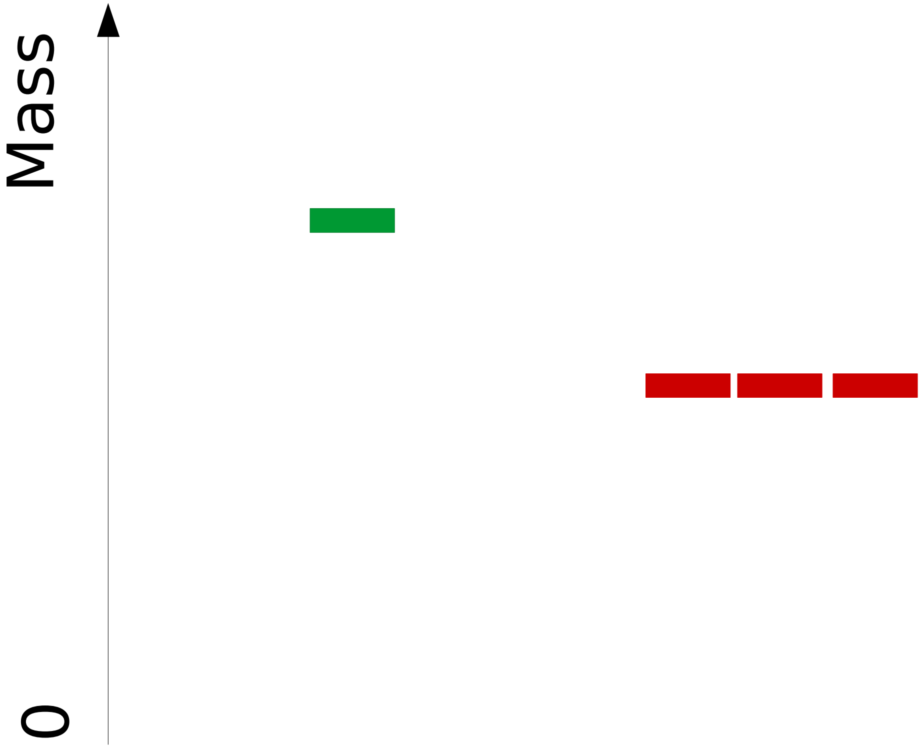
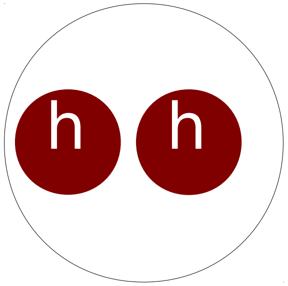


Custodial singlet

$$h(x)^{+}\,h(x)$$



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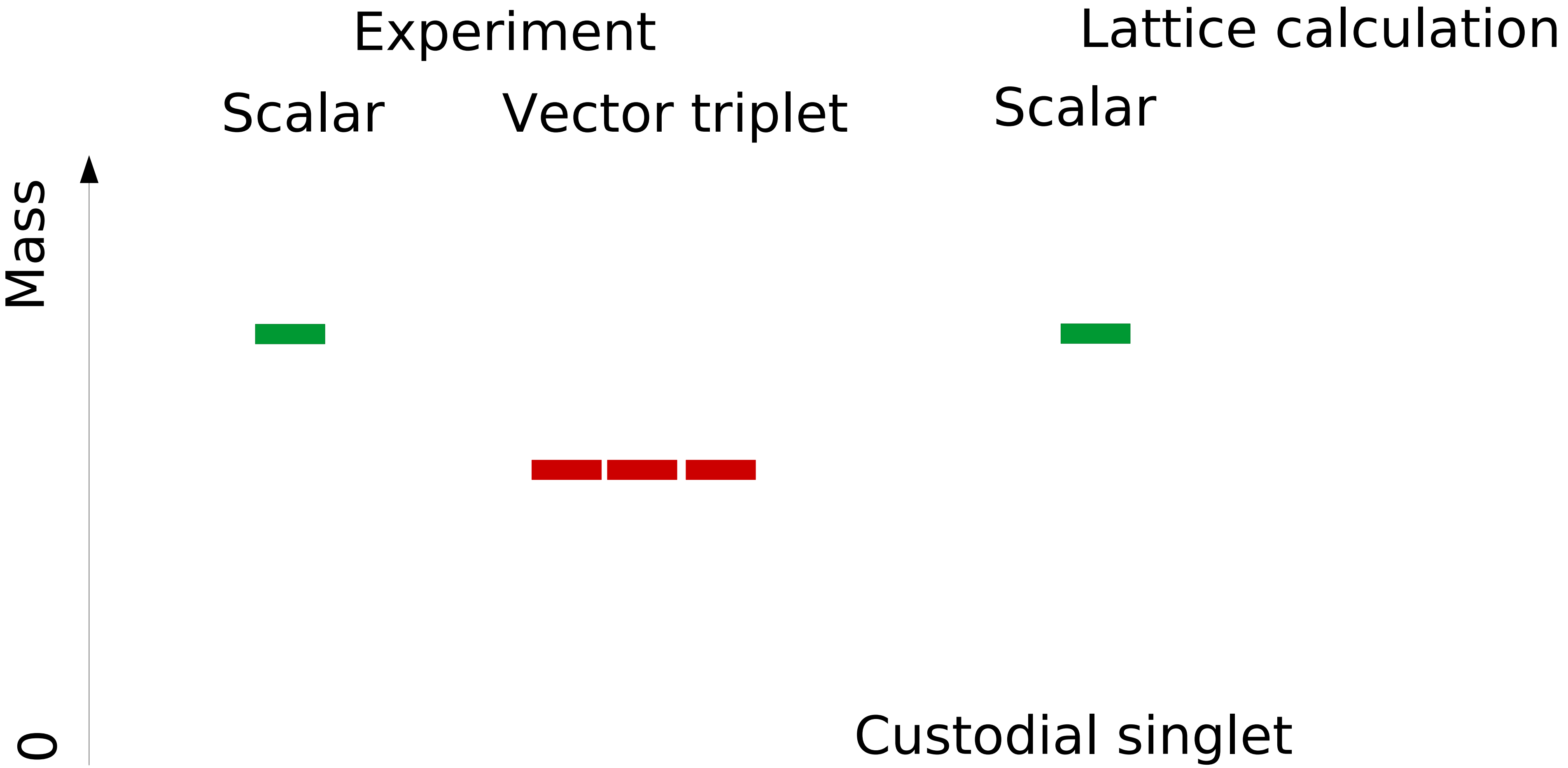

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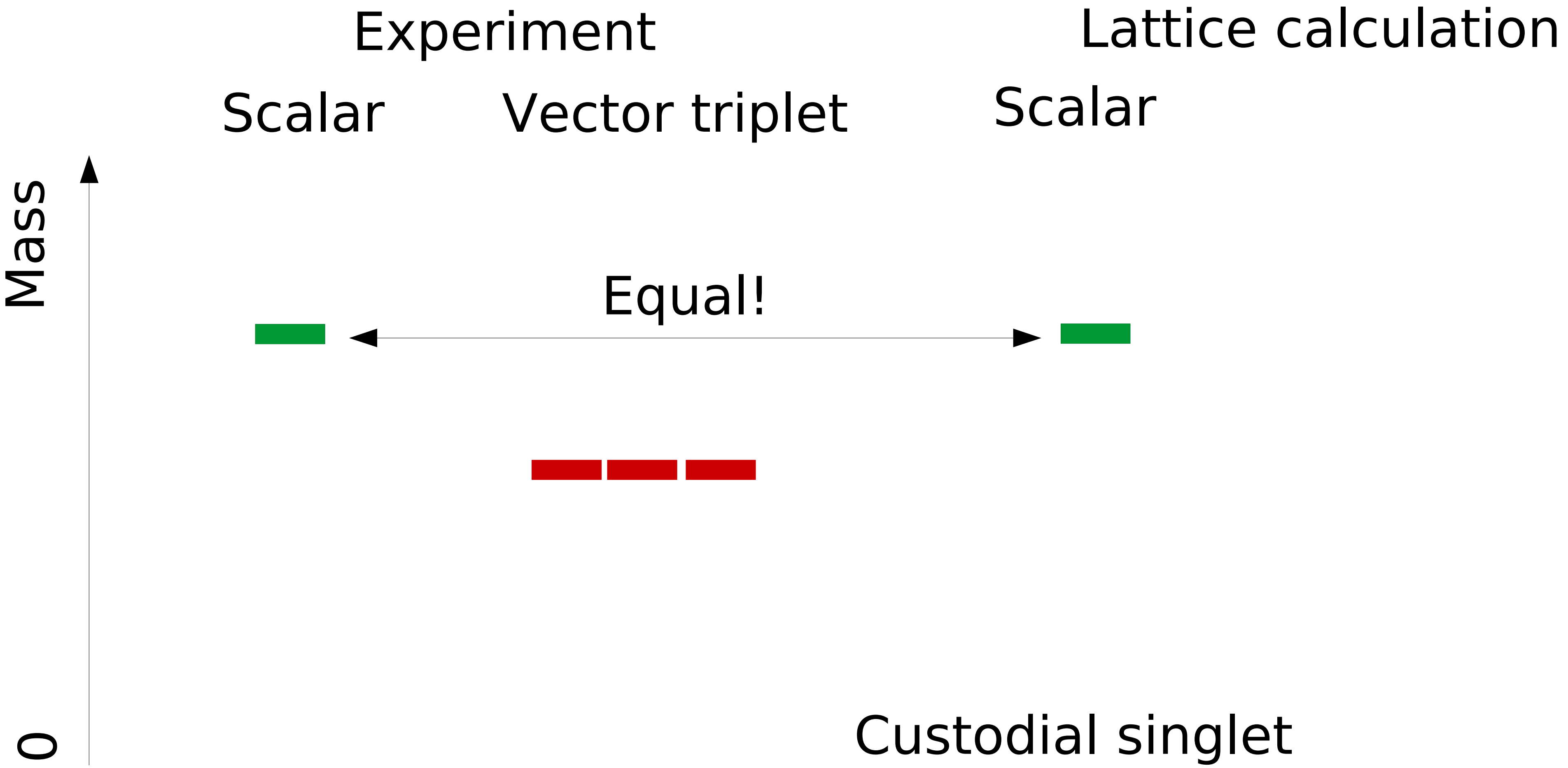

Experiment

Lattice calculation

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Mass

Equal!

0

Custodial singlet

How to make predictions

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  - Perform double expansion [Fröhlich et al.'80, Maas’12]
    - Vacuum expectation value (FMS mechanism)
    - Standard expansion in couplings
    - Together: Gauge-invariant perturbation theory

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Bound state mass

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0^+ singlet: $\langle (h^+ h)(x)(h^+ h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

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3) Standard perturbation theory

Bound state mass

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Scattering state

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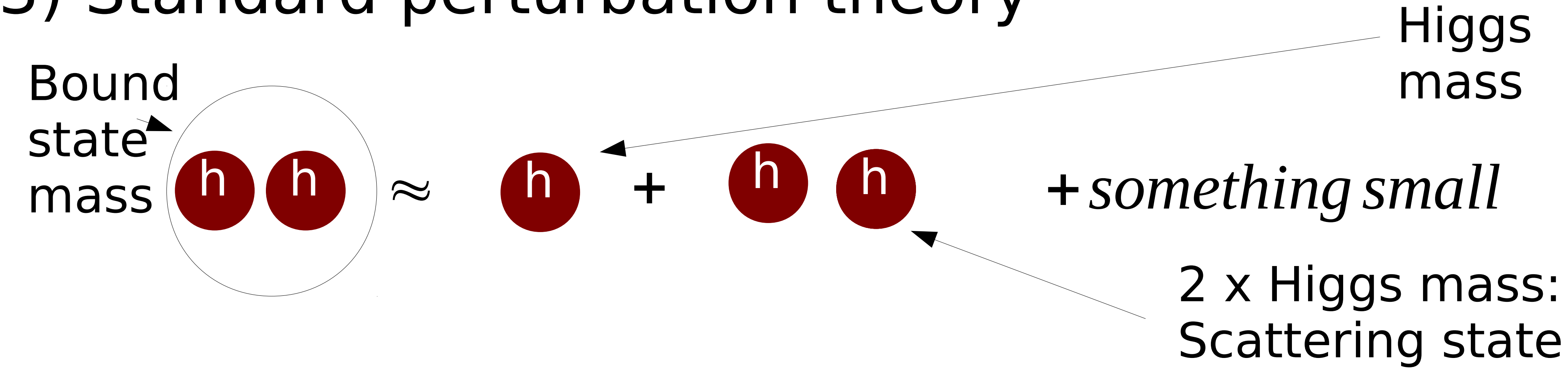

Bound state mass $\approx$ + $+ \mathit{something\ small}$

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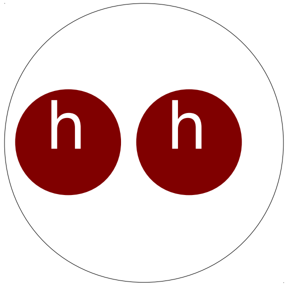
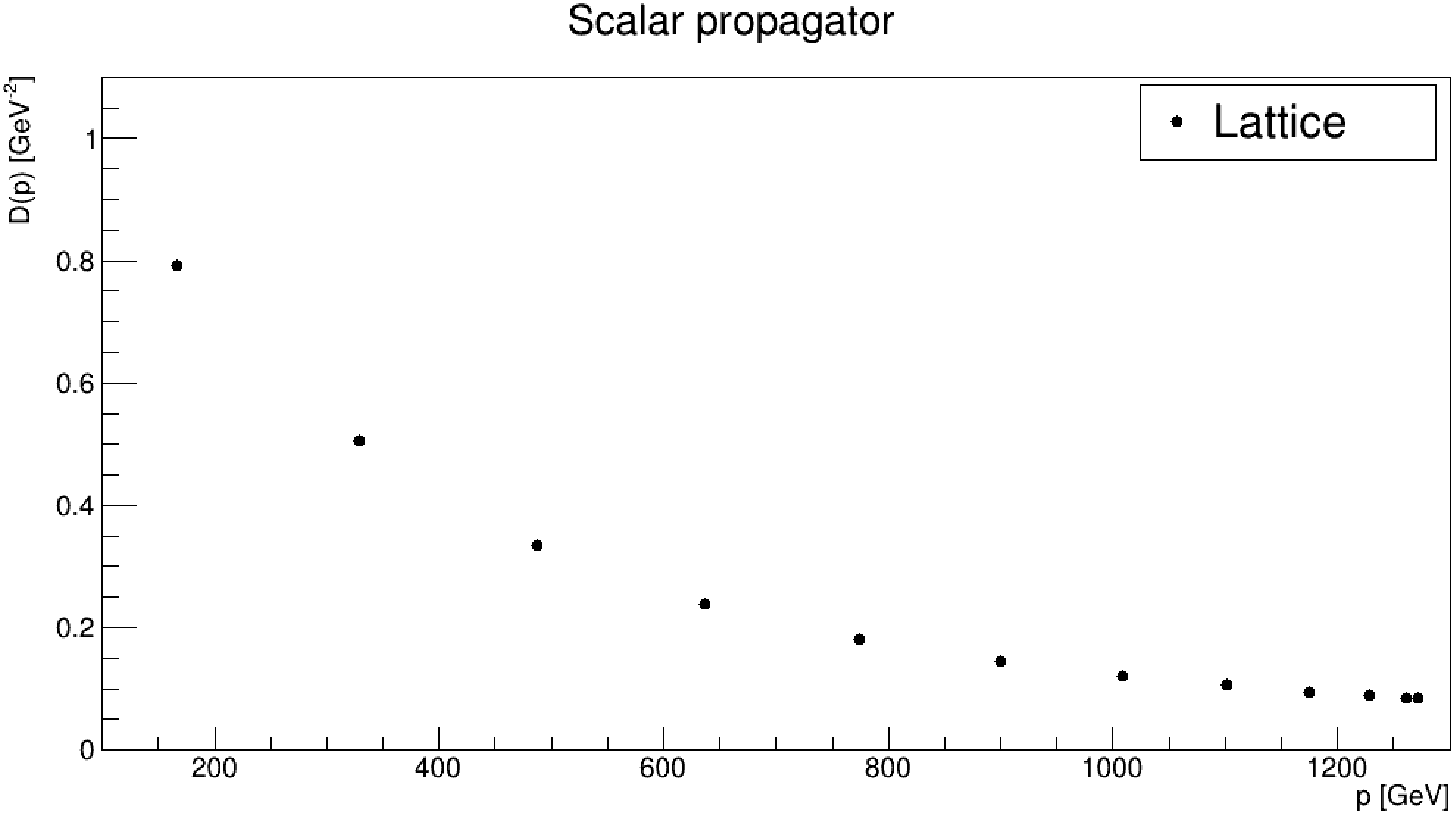
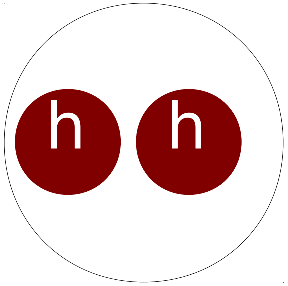


$$= D\left(P^2\right)$$

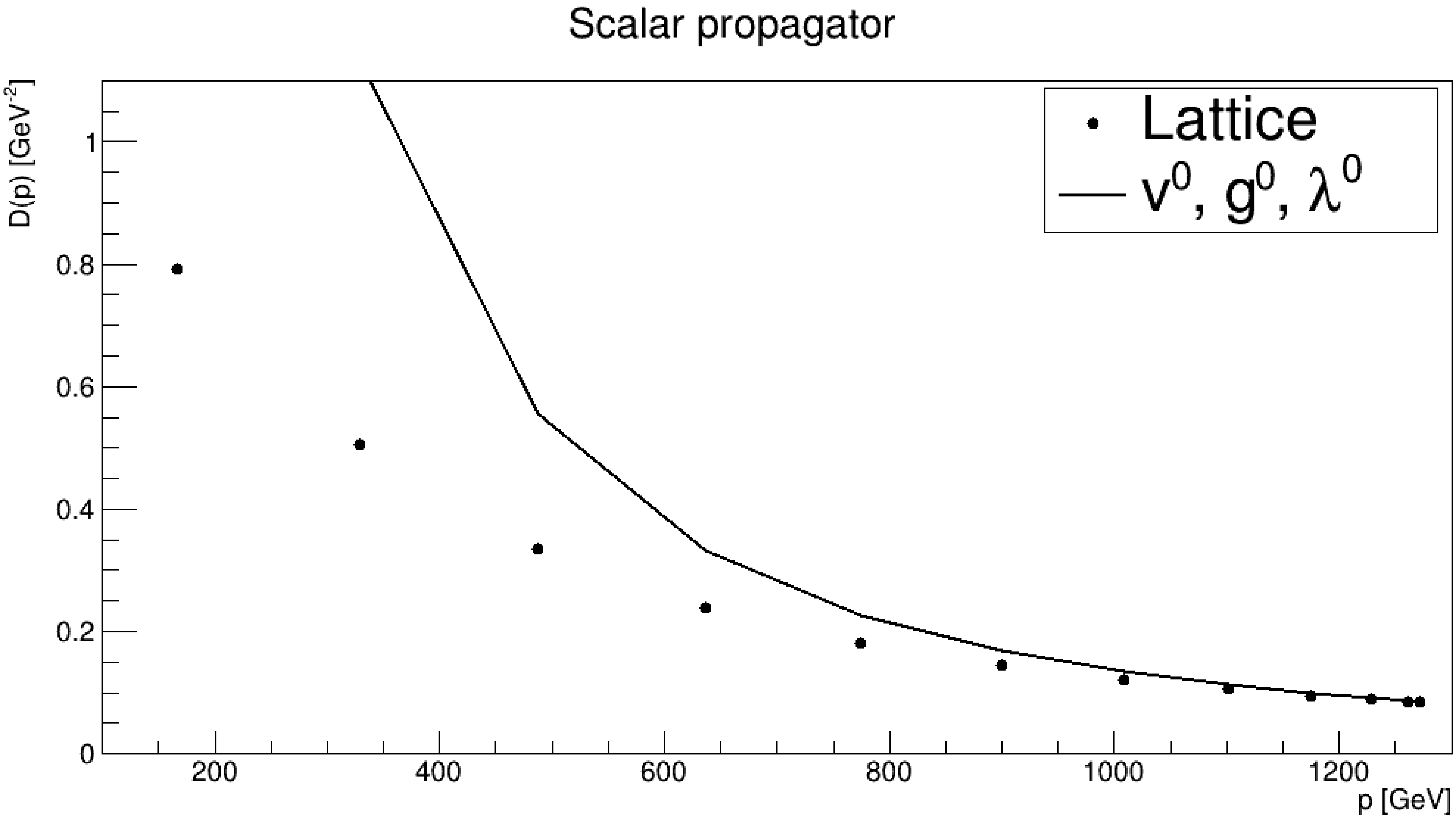
Gauge-invariant perturbation theory

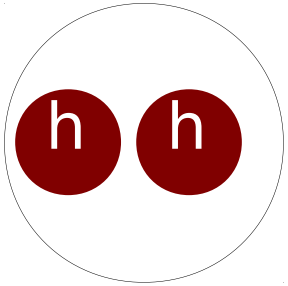




$$D(P) = \langle (h^+ h)(x)(h^+ h)(y) \rangle$$

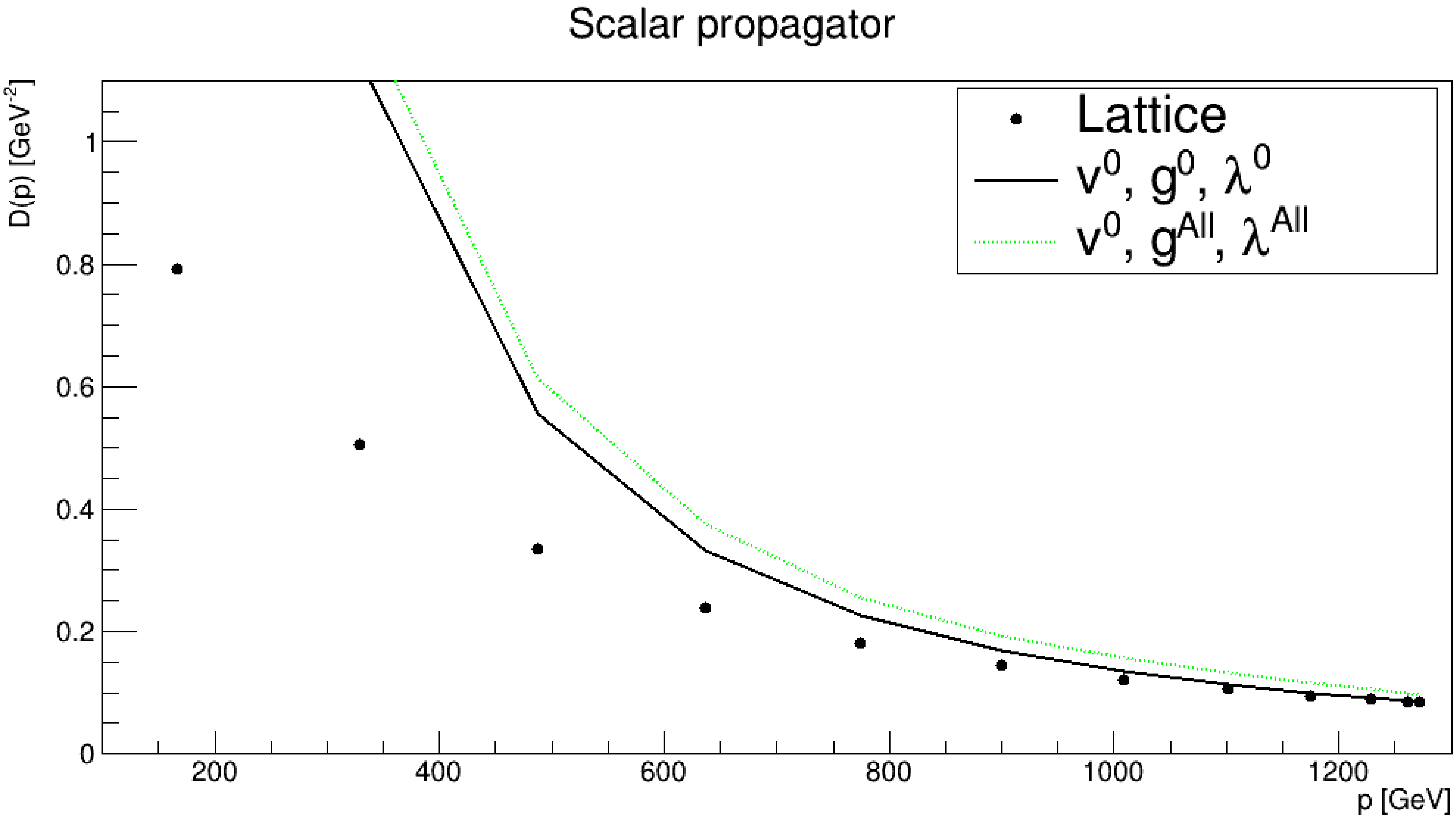
Gauge-invariant perturbation theory

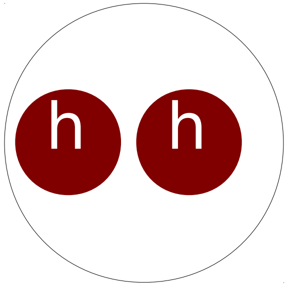

Scalar propagator



$$D(P)=\langle (h^+ h)(x)(h^+ h)(y)\rangle$$
$$=v^2\langle \eta^+ (x)\eta(y)\rangle_{tree\,level}$$

Gauge-invariant perturbation theory





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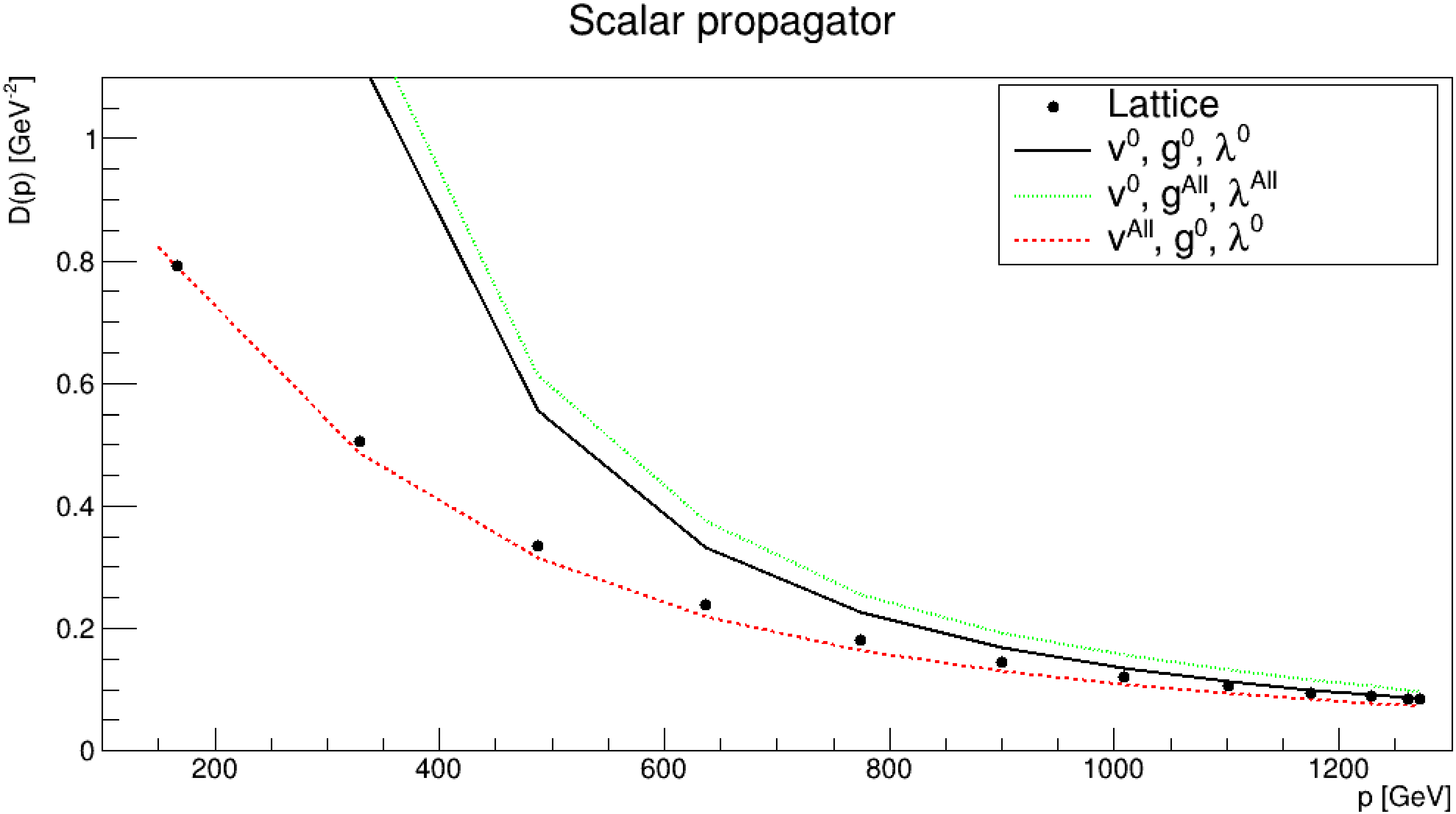

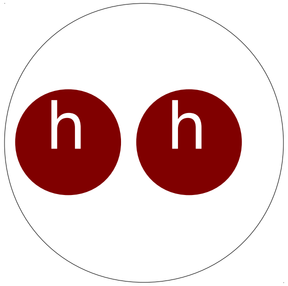

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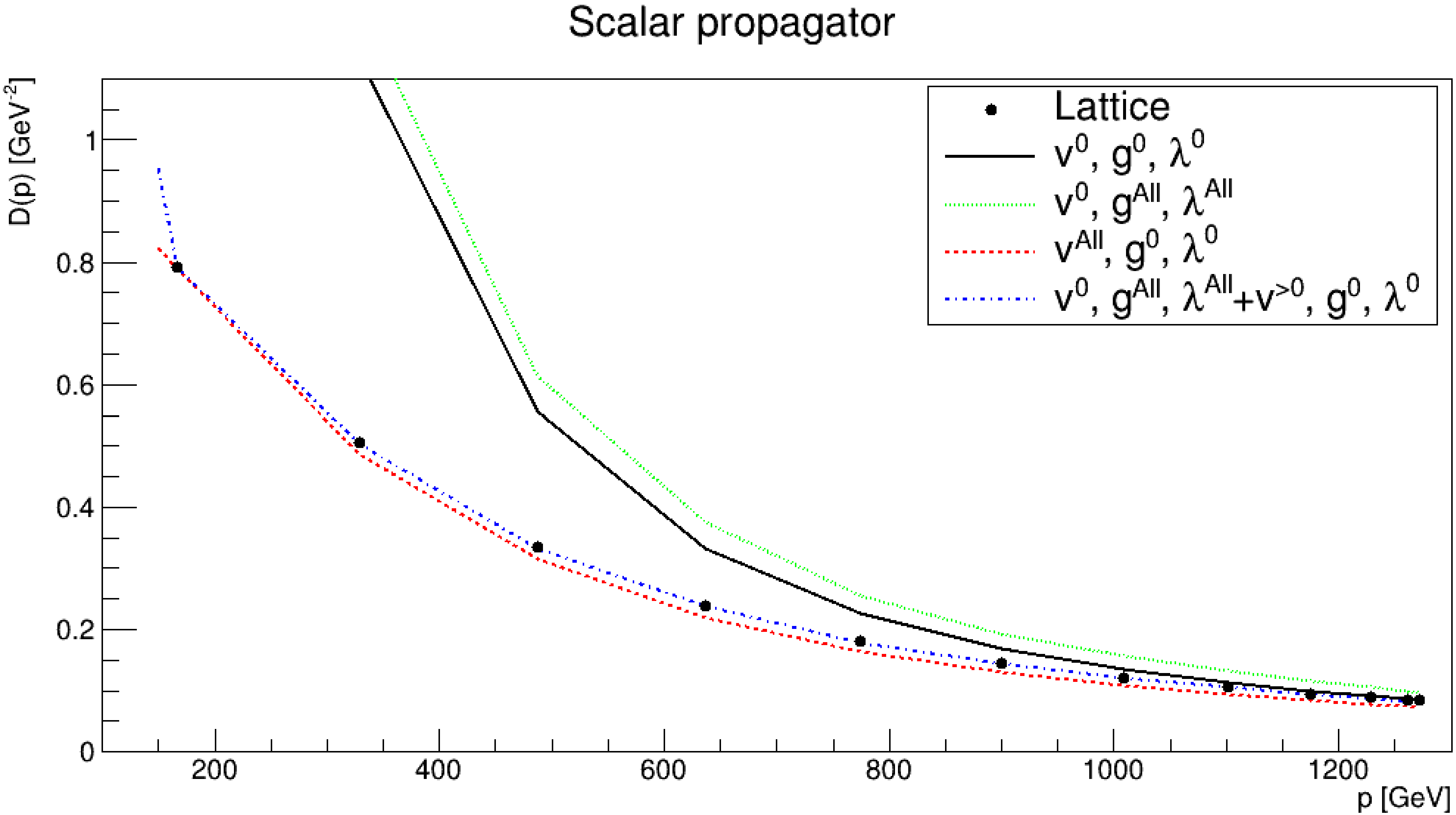

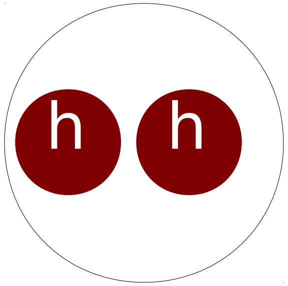

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What about the vector?

[Fröhlich et al.’80,’81
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Matrix from
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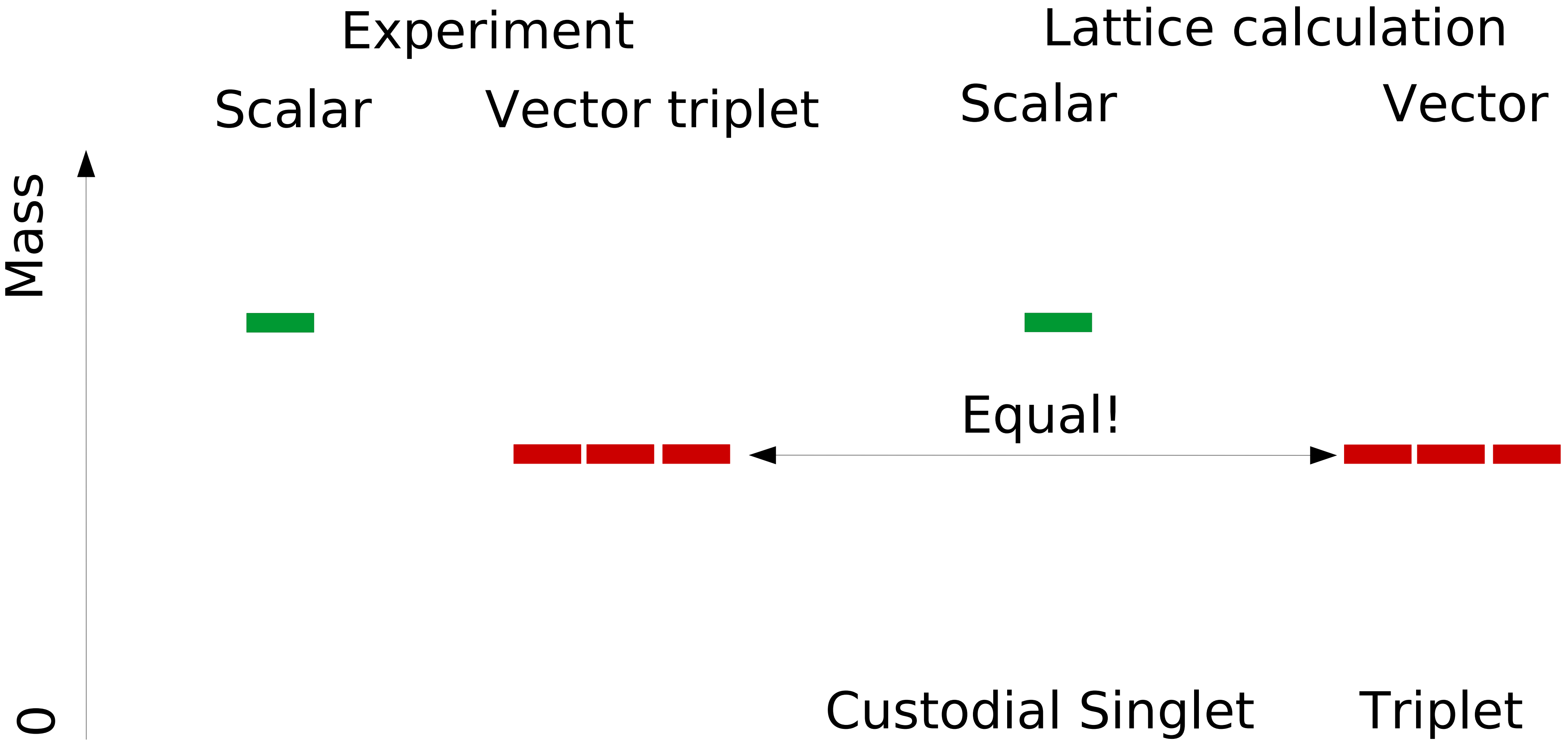
Exactly one gauge boson
for every physical state

Physical spectrum

[Maas'12, Maas & Mufti'14]



Experiment

Lattice calculation

Scalar

Vector triplet

Scalar

Vector

Mass

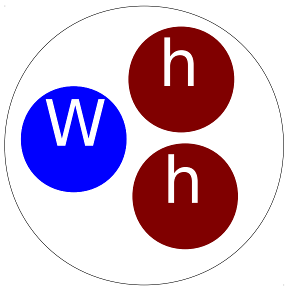
0

Equal!

Custodial Singlet

Triplet

$$\mathrm{tr}\, t^{\textcircled{a}} \frac{h^{+}}{\sqrt{h^{+} h}} D_{\mu} \frac{h}{\sqrt{h^{+} h}}$$



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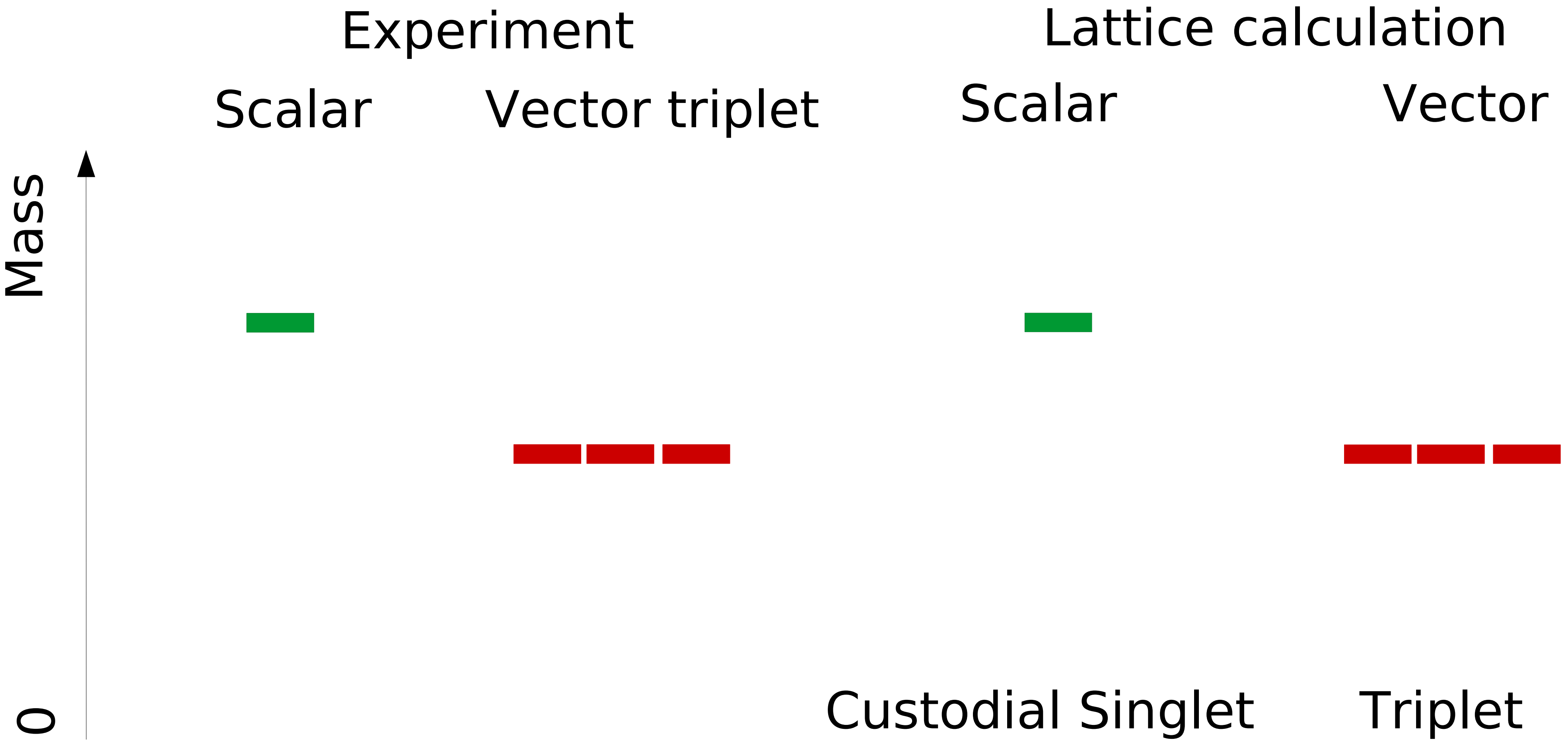
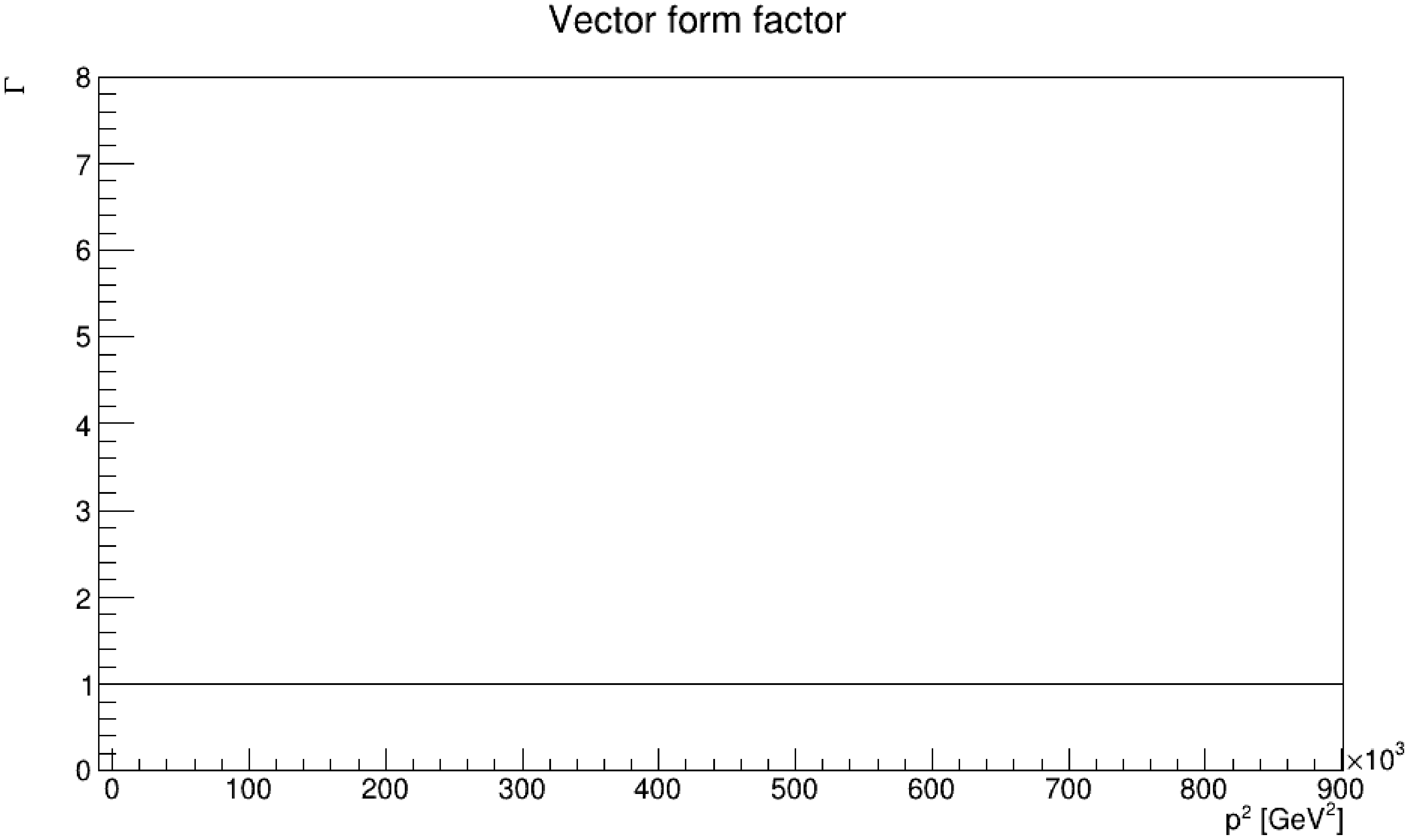


- FMS works
  - Some lattice support for SU(2)xU(1) [Shrock et al. 85-88]
  - Extension to the whole standard model

Bound states as extended objects

[Maas,Raubitzke,Törek'18]



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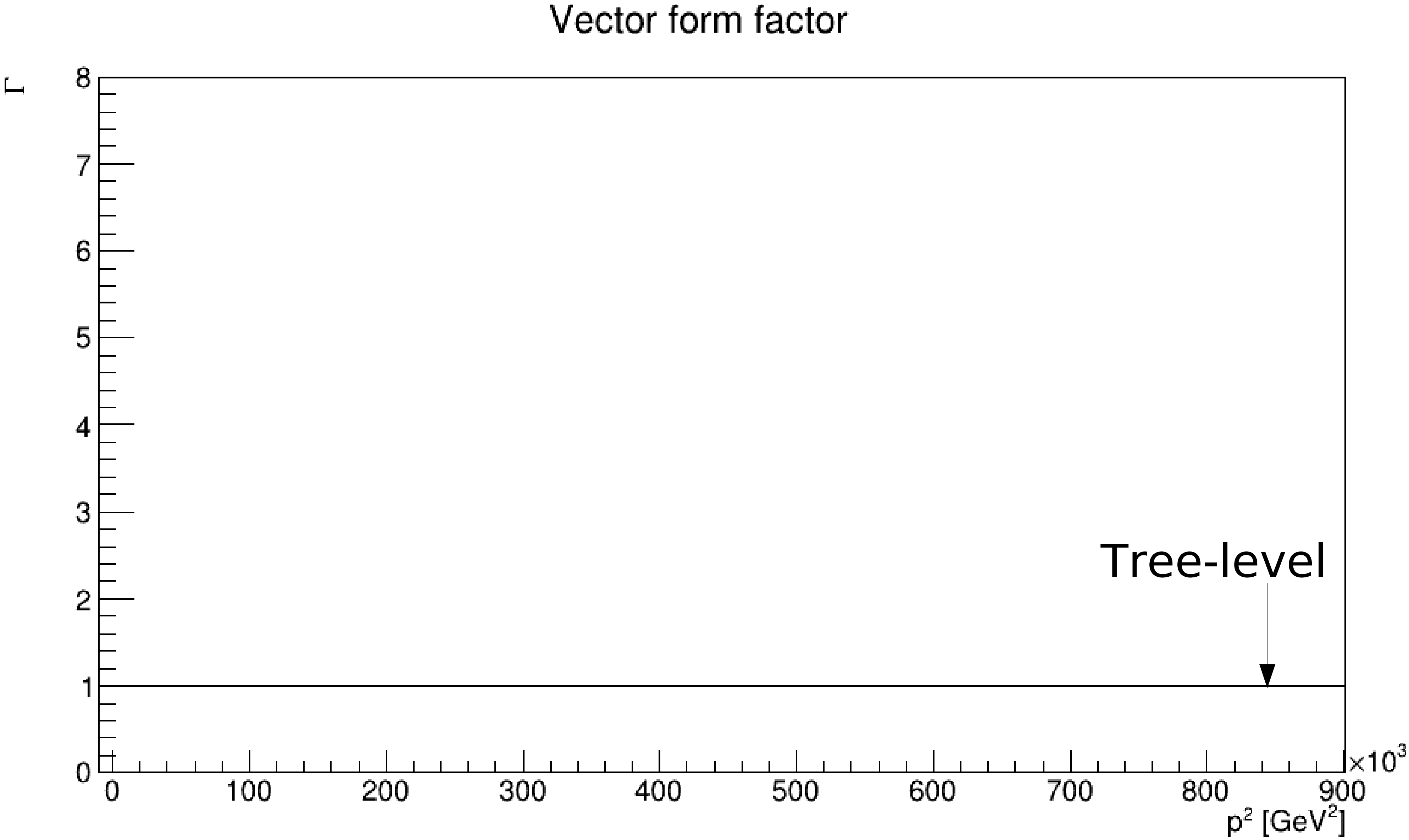

Tree-level

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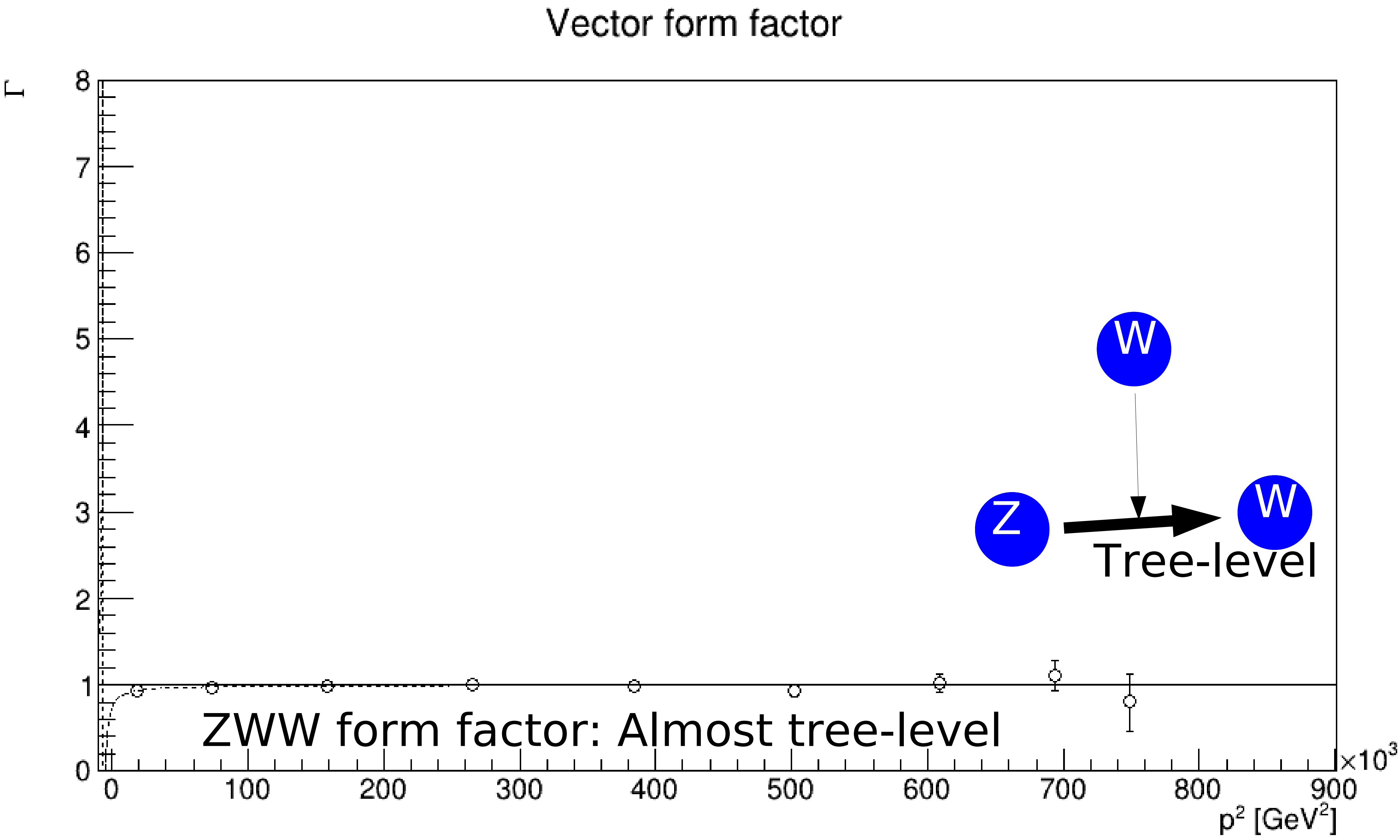

Tree-level

ZWW form factor: Almost tree-level

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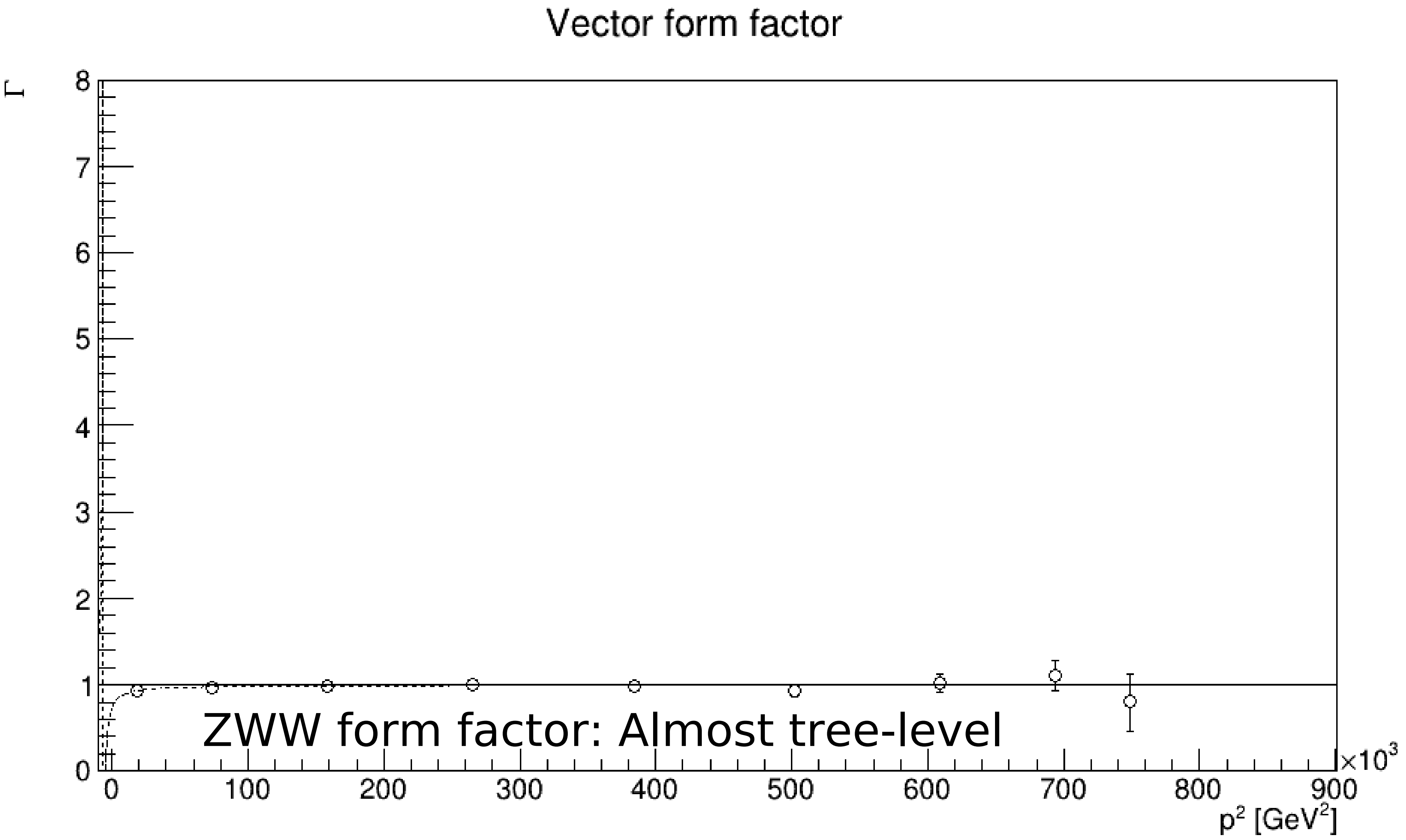

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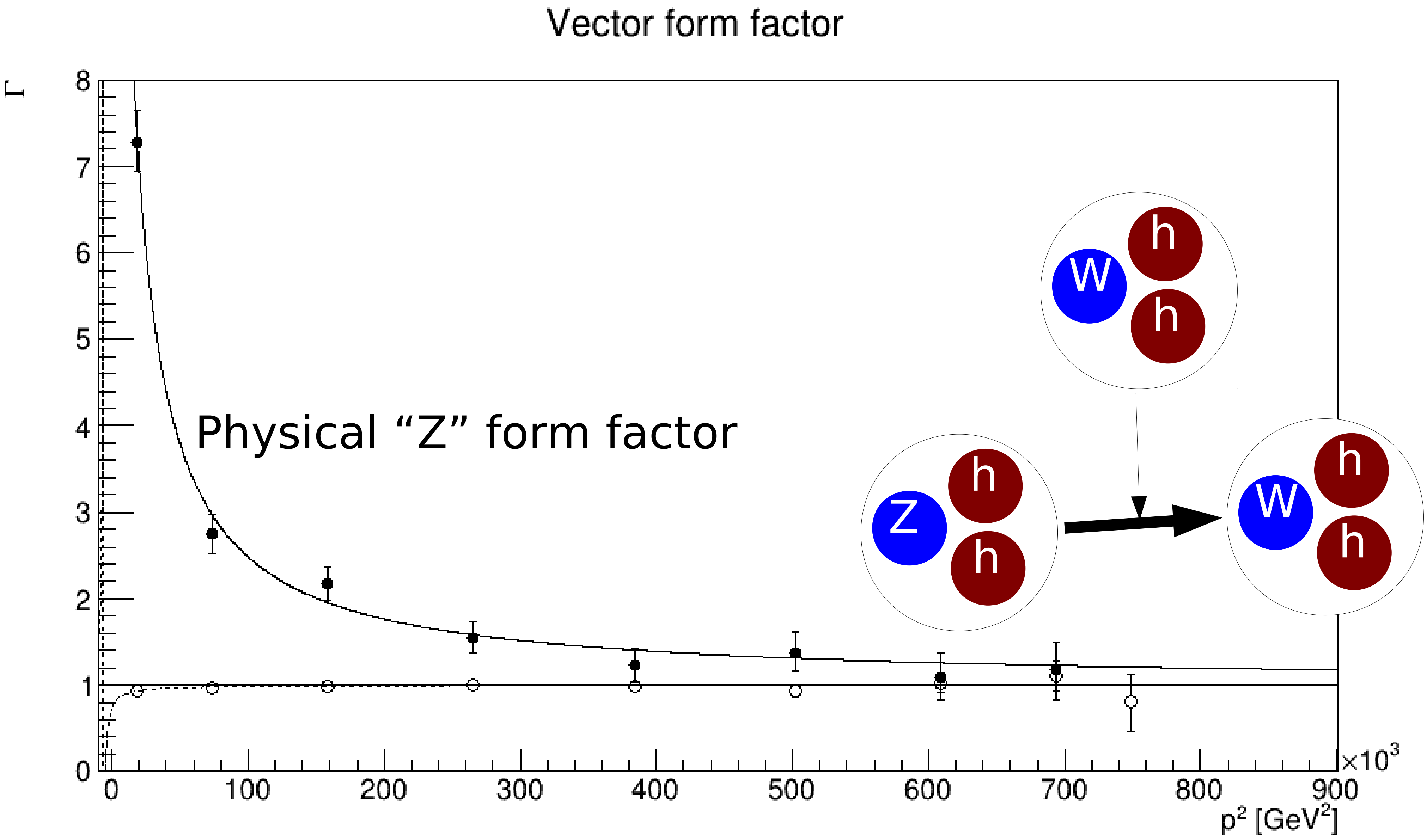

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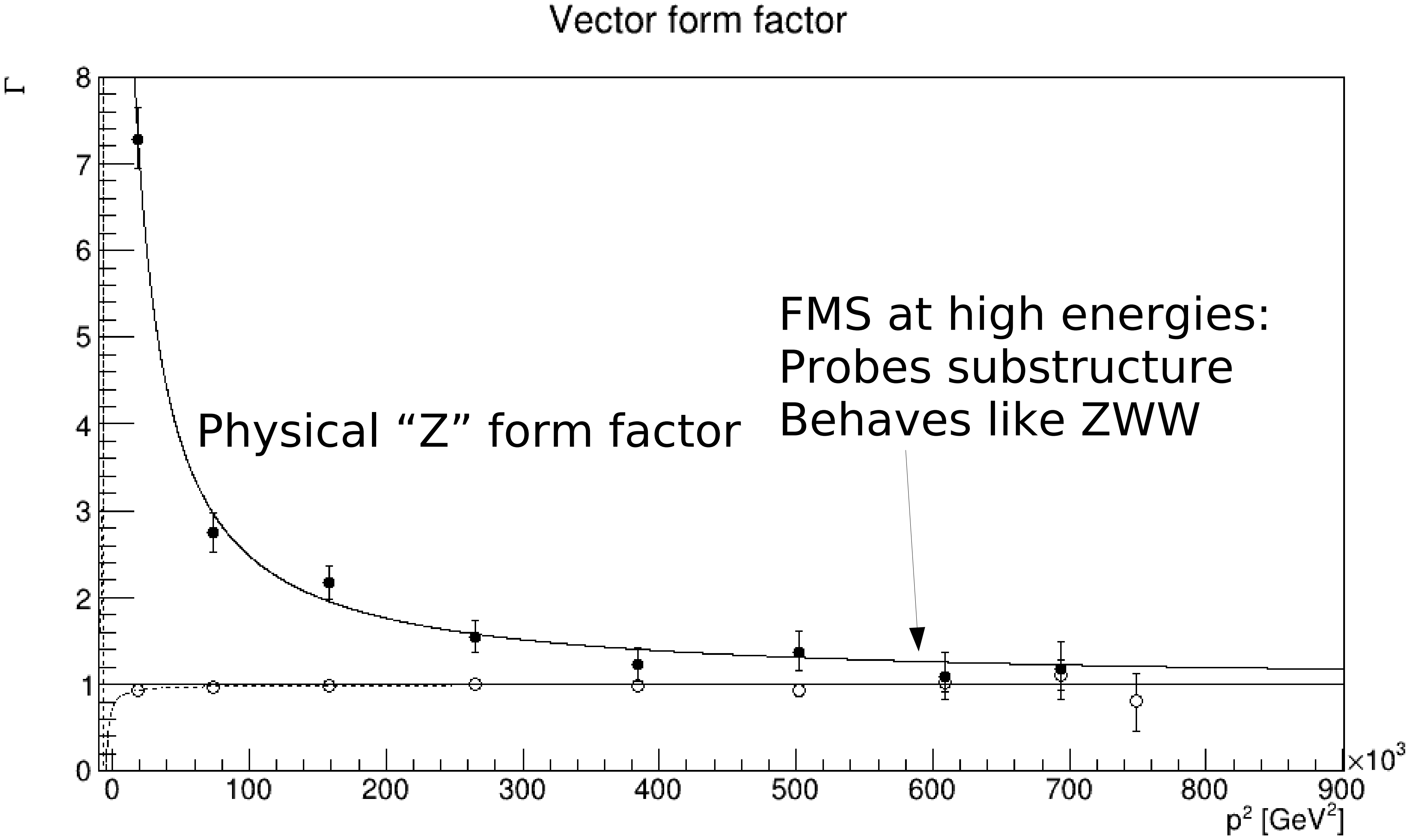

Physical “Z” form factor

FMS at high energies:
Probes substructure
Behaves like ZWW

Bound states as extended objects

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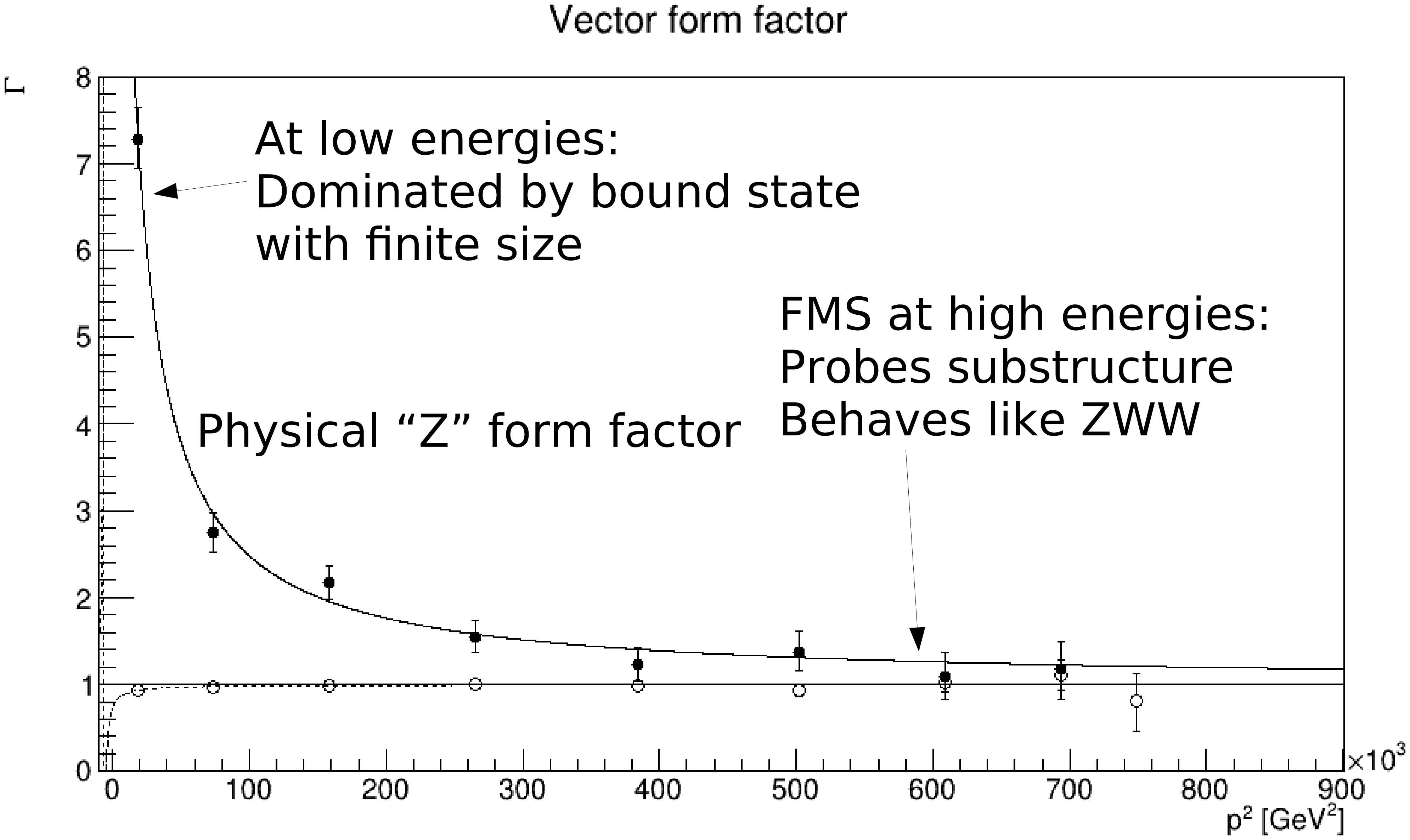

At low energies:
Dominated by bound state
with finite size

Physical "Z" form factor

FMS at high energies:
Probes substructure
Behaves like ZWW

- Physical "Z" *mr~2*

Beyond quantitative

- FMS worked because corrections are small
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- What happens if there are qualitative effects?
  - Different structures of local and global symmetry
- FMS still works, if quantum fluctuations in a suitable gauge are small: Example

A toy model

- Consider an SU(3) with a single fundamental scalar

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- Consider an SU(3) with a single fundamental scalar
- Looks very similar to the standard model Higgs

$$L=-\frac{1}{4}W^{a}_{\mu\nu}W^{\mu\nu}_{a}$$

$$W^{a}_{\mu\nu}=\partial_{\mu}W^{a}_{\nu}-\partial_{\nu}W^{a}_{\mu}+gf^{a}_{bc}W^{b}_{\mu}W^{c}_{\nu}$$

- Ws W^{a}_{μ} Ⓦ

- Coupling g and some numbers f^{abc}

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- Couplings g, v, λ and some numbers f^{abc} and t_{a}^{ij}
- Parameters selected for a BEH effect

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- Global U(1) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only

$$W_{\mu}^{a}\rightarrow W_{\mu}^{a} \qquad h\rightarrow \exp(ia)h$$

Textbook approach

- Choose parameters to get a Brout-Englert-Higgs effect

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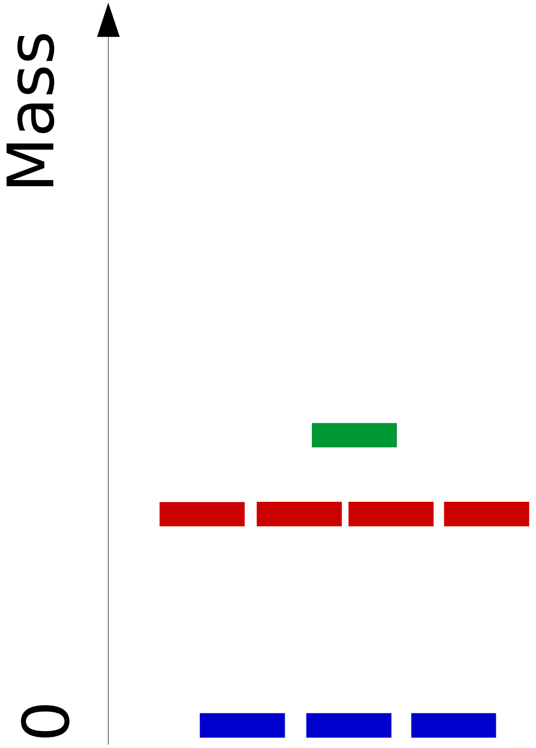
- Perform perturbation theory

Spectrum

Gauge-dependent

Vector



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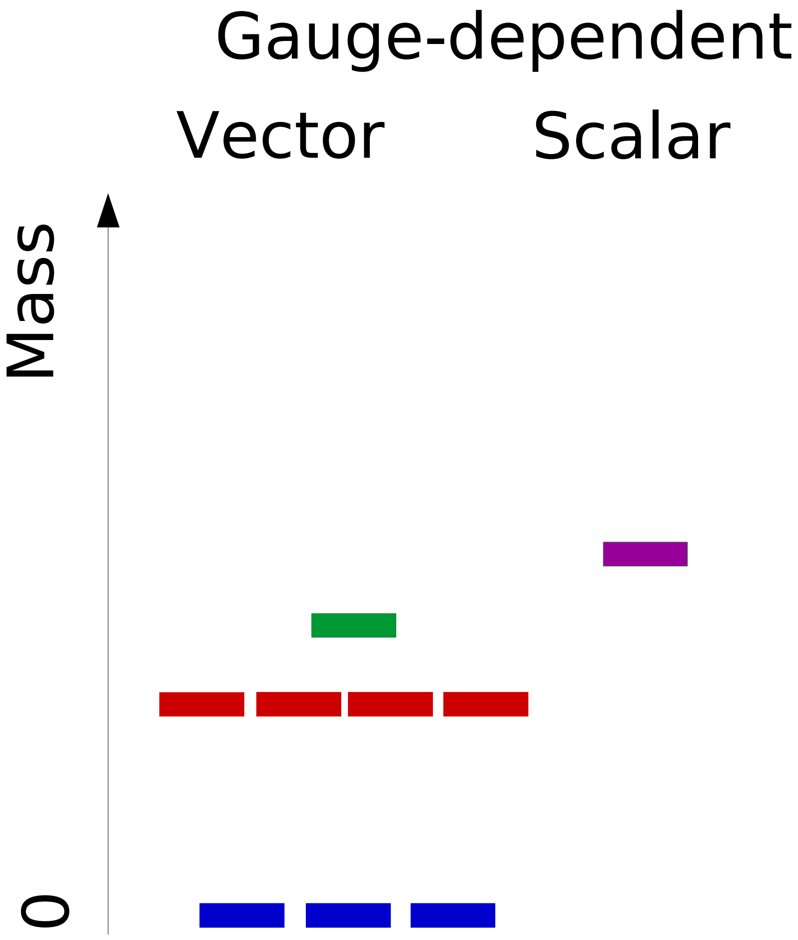
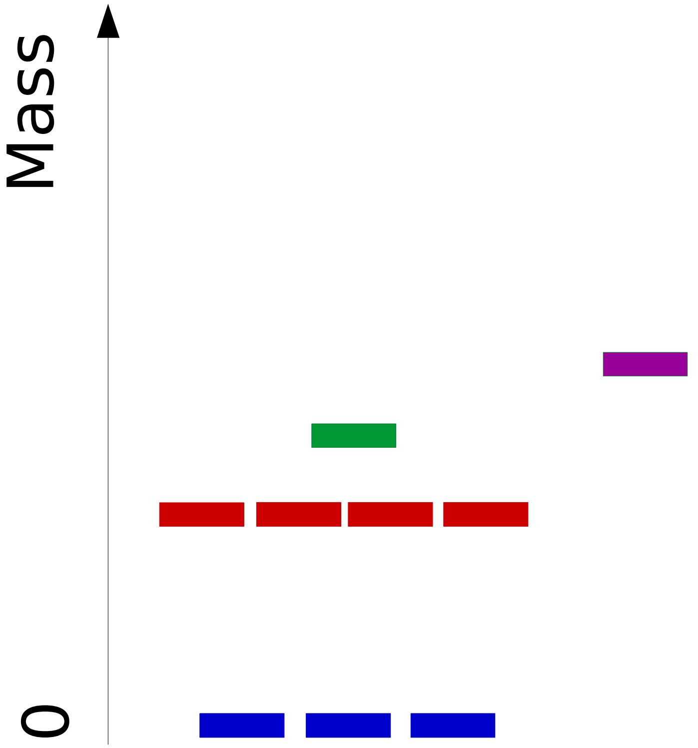


Spectrum

[Maas & Törek’16,'18
Maas, Sondenheimer & Törek'17]

Gauge-dependent

Vector Scalar



Spectrum

[Maas & Török’16,'18
Maas, Sondenheimer & Török'17]

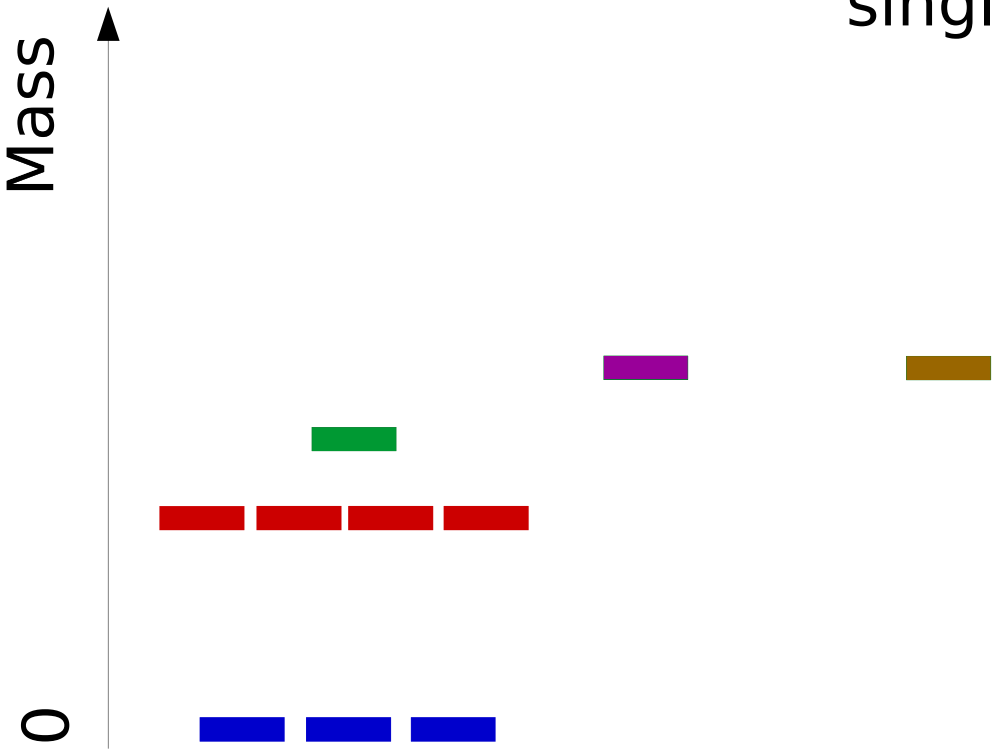
Gauge-dependent

Gauge-invariant

Vector

Scalar

Scalar singlet



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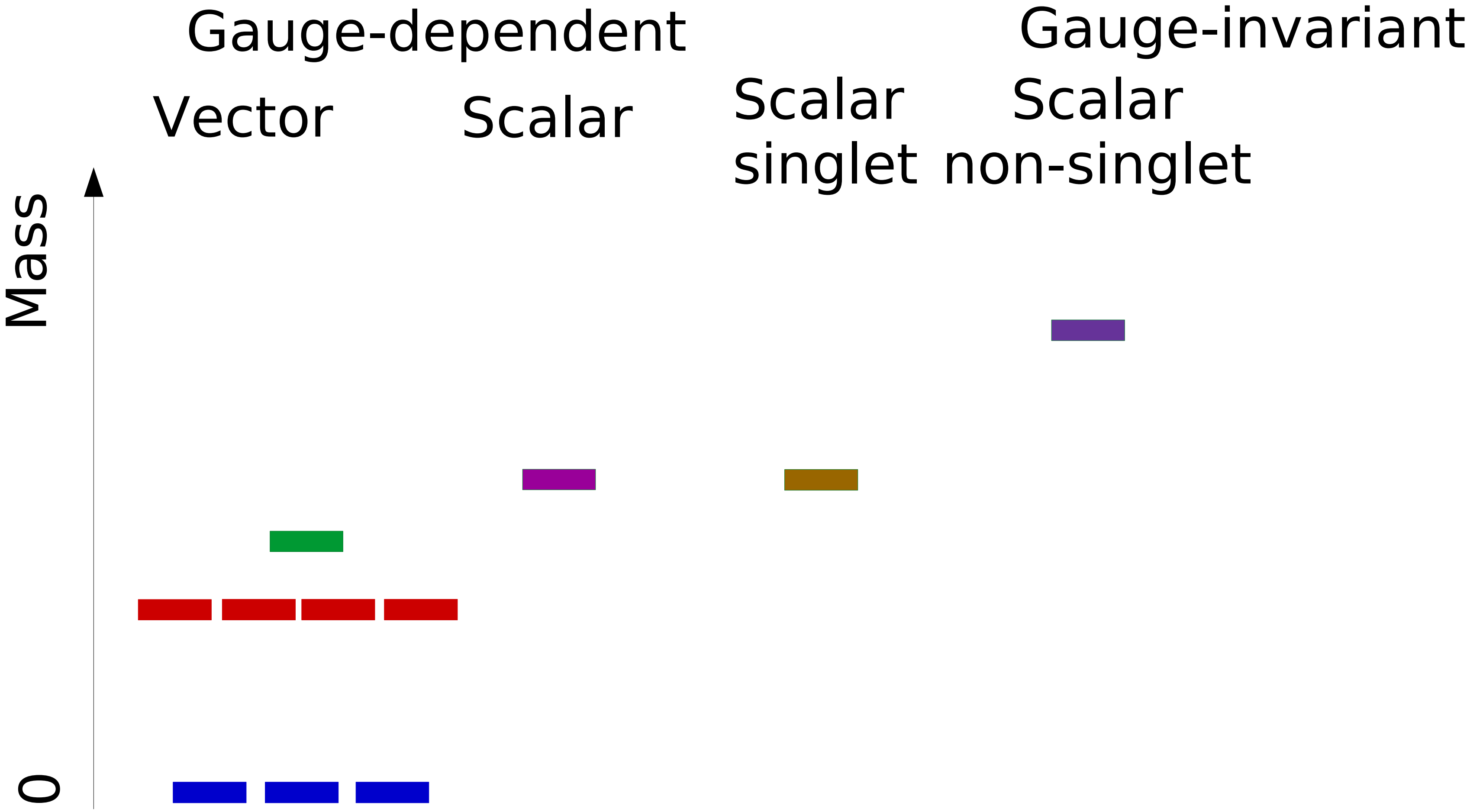

Gauge-dependent

Gauge-invariant

Vector

Scalar

Scalar
singlet

Scalar
non-singlet

Spectrum

[Maas & Törek’16,'18
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Gauge-dependent

Vector

Scalar

Gauge-invariant

Scalar singlet

Scalar non-singlet

Vector singlet

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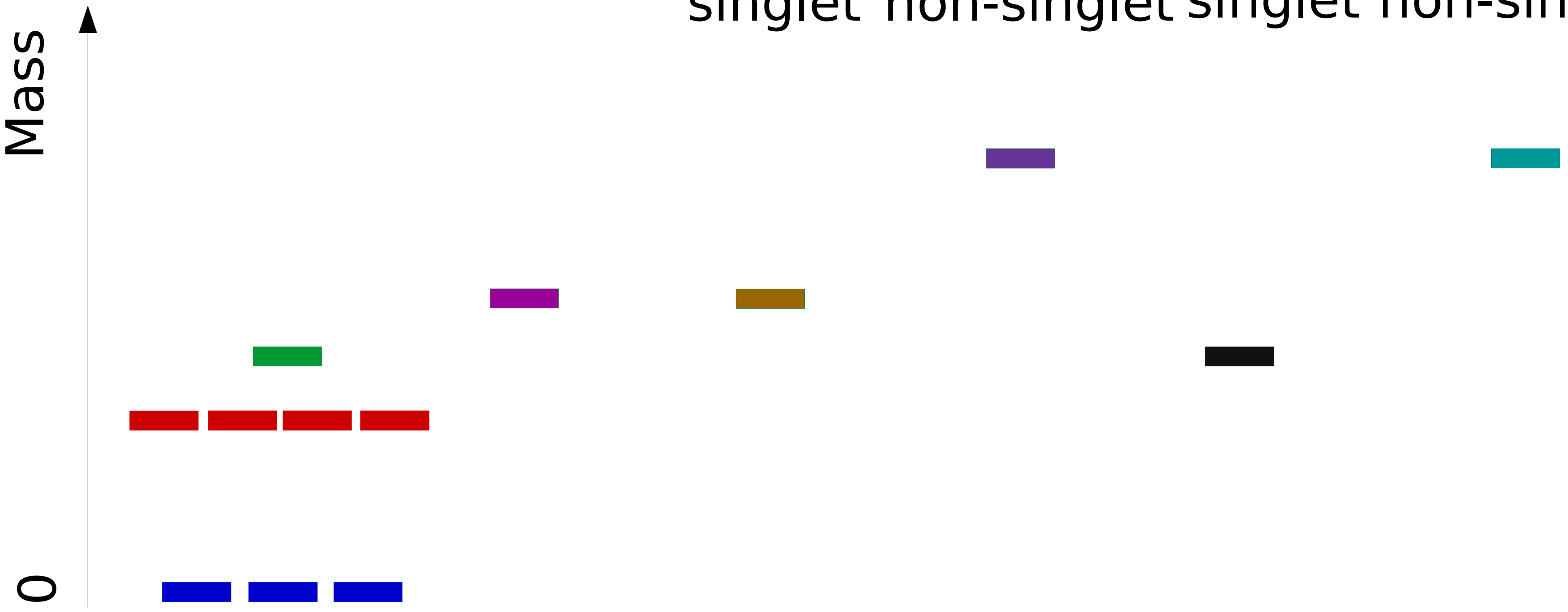

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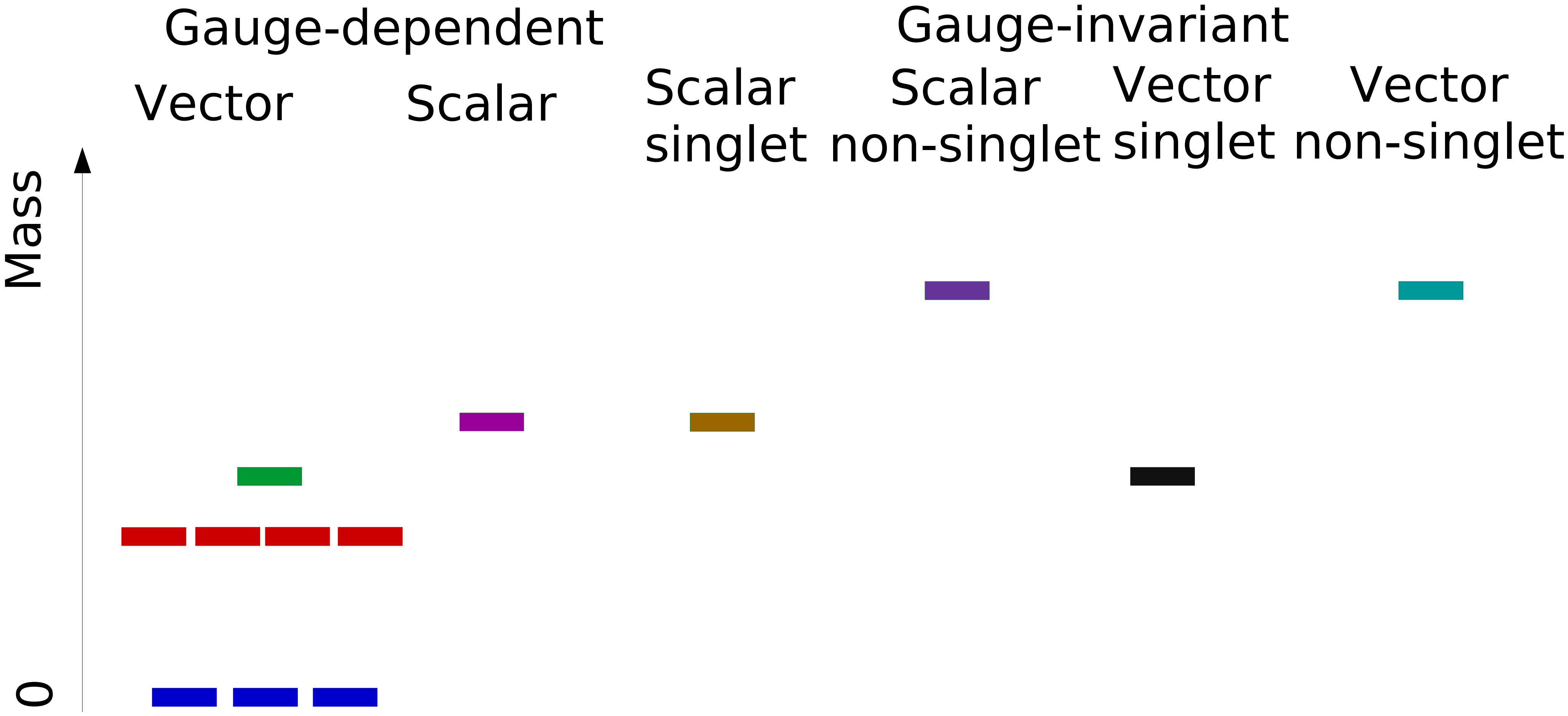


Gauge-dependent

Gauge-invariant

Vector

Scalar

Scalar singlet

Scalar non-singlet

Vector singlet

Vector non-singlet

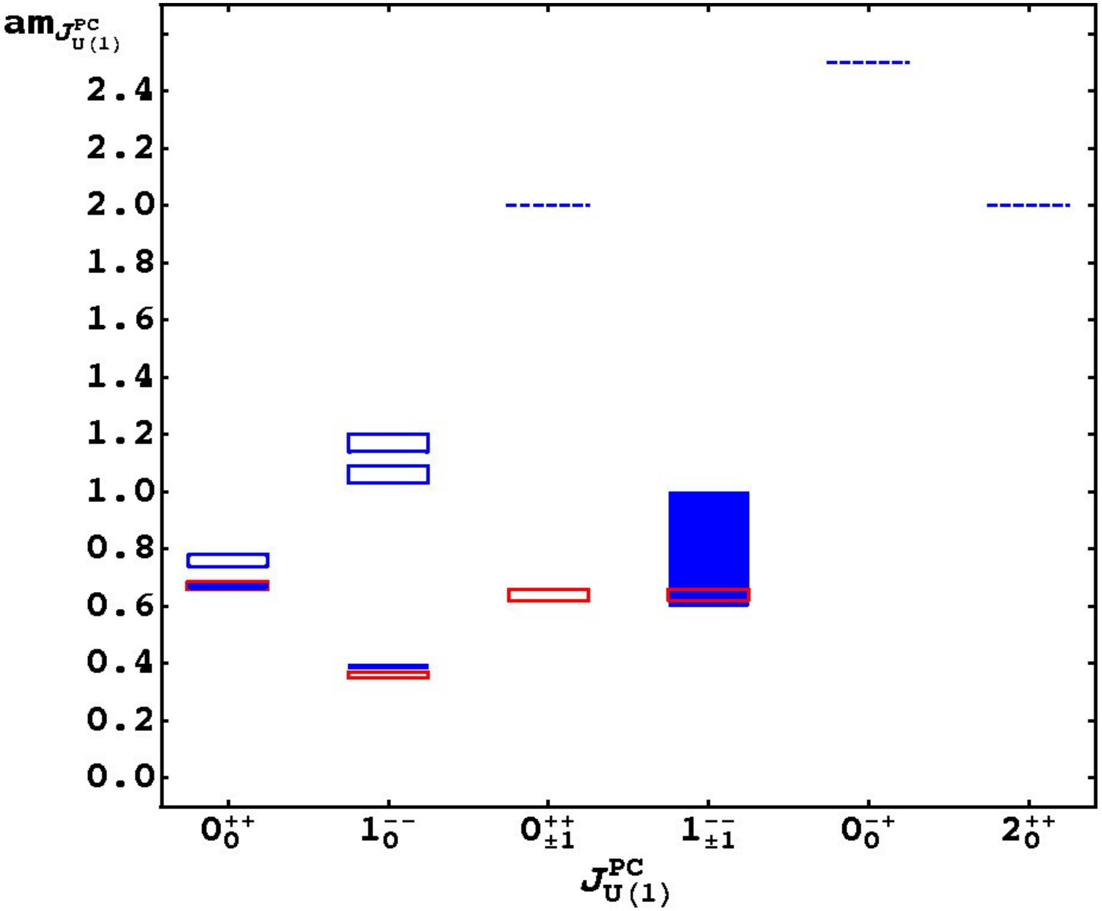
- Qualitatively different spectrum

Spectrum

[Maas & Török’16,'18
Maas, Sondenheimer & Török'17]

Gauge-invariant

Scalar singlet | Scalar non-singlet | Vector singlet | Vector non-singlet



- Qualitatively different spectrum

Spectrum

[Maas & Török'16,'18
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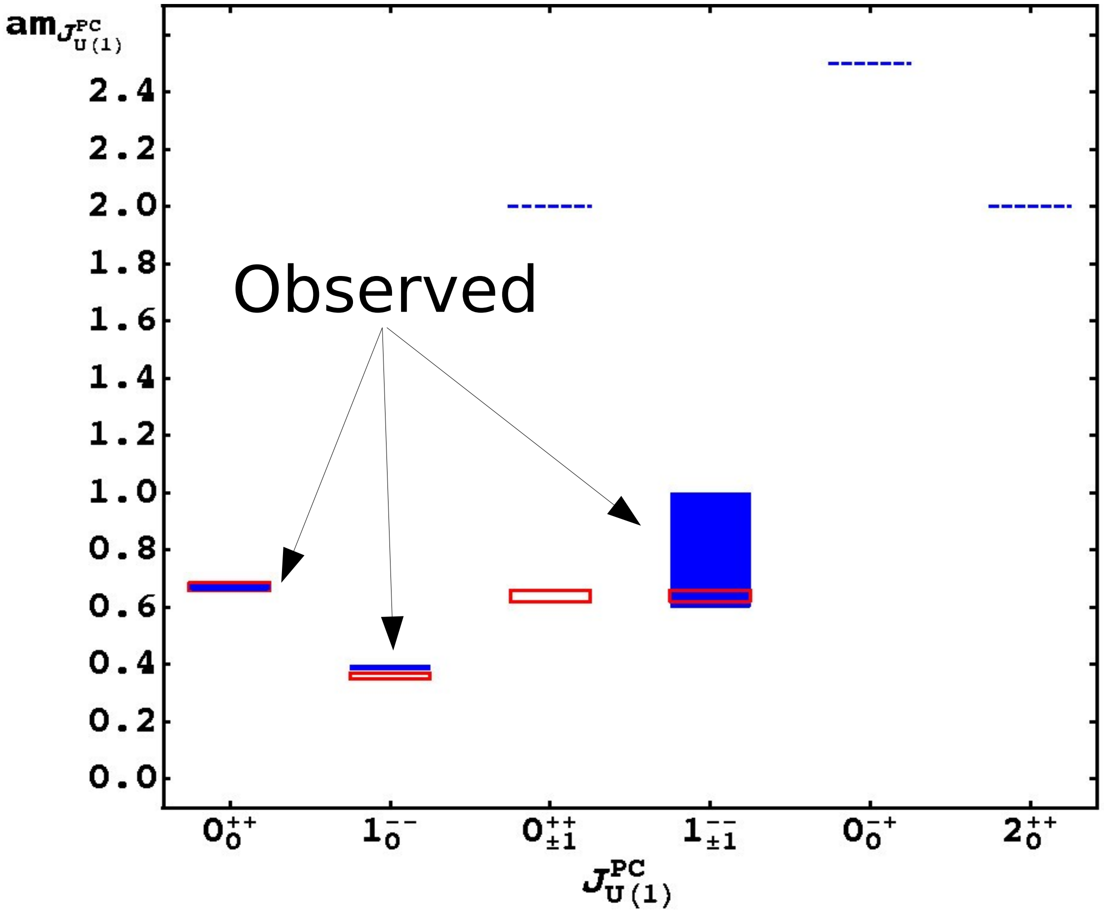

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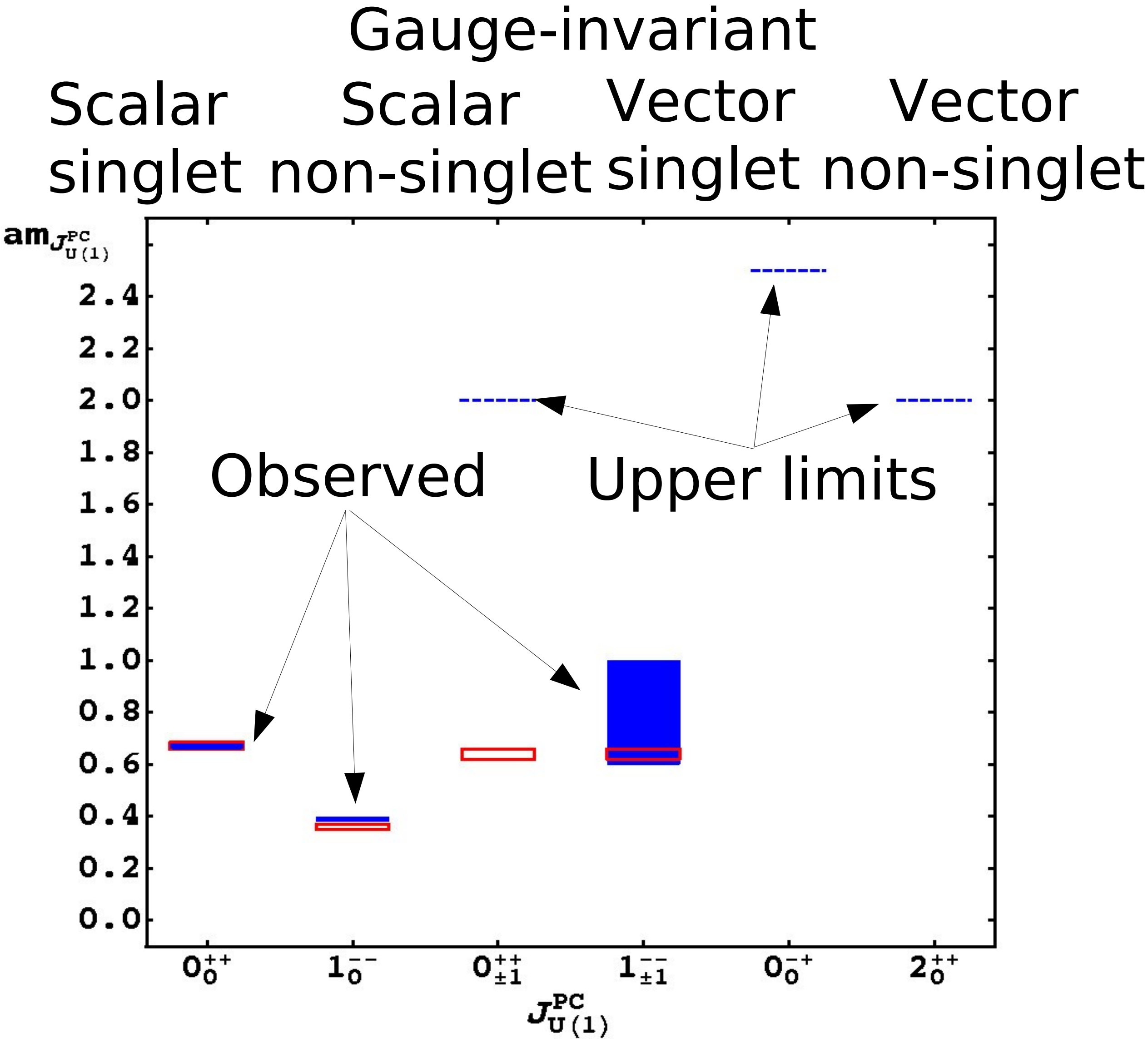

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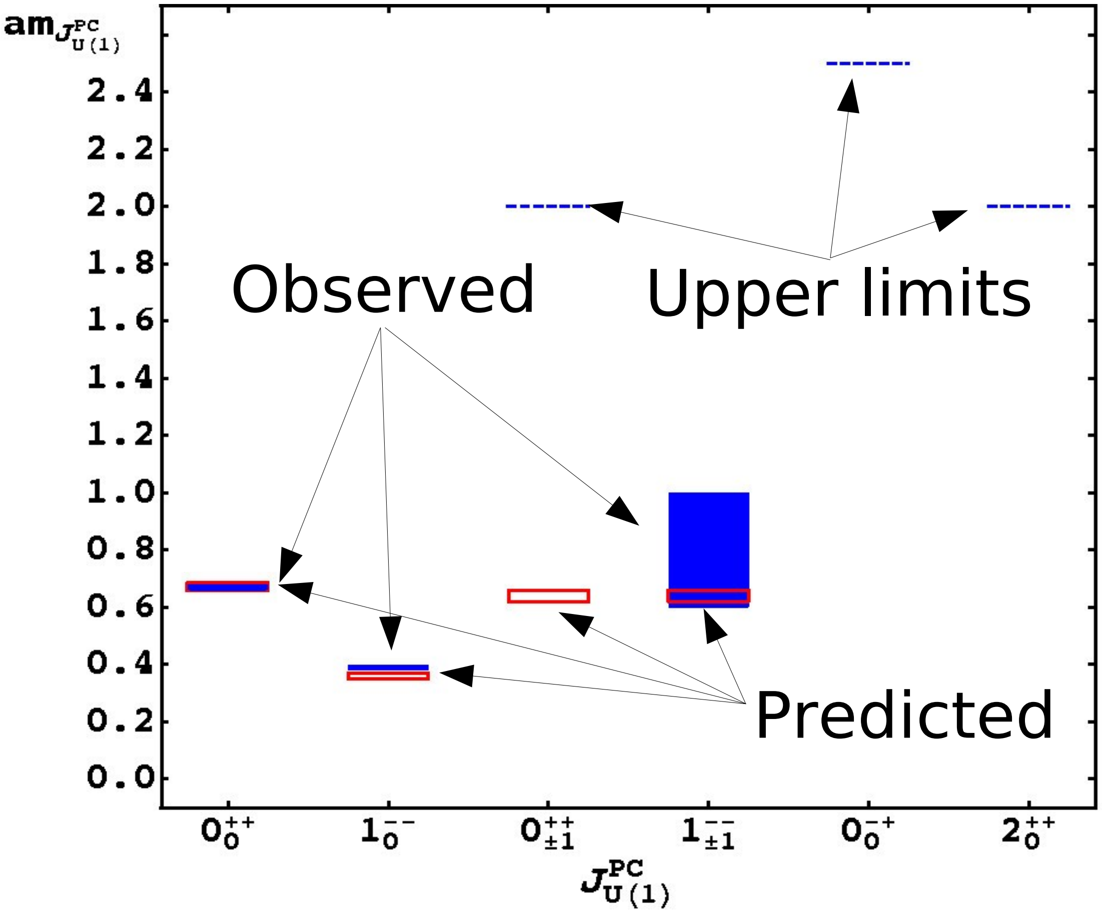

Observed

Upper limits

Predicted

- Qualitatively different spectrum
- Results in agreement with analytic predictions

Spectrum

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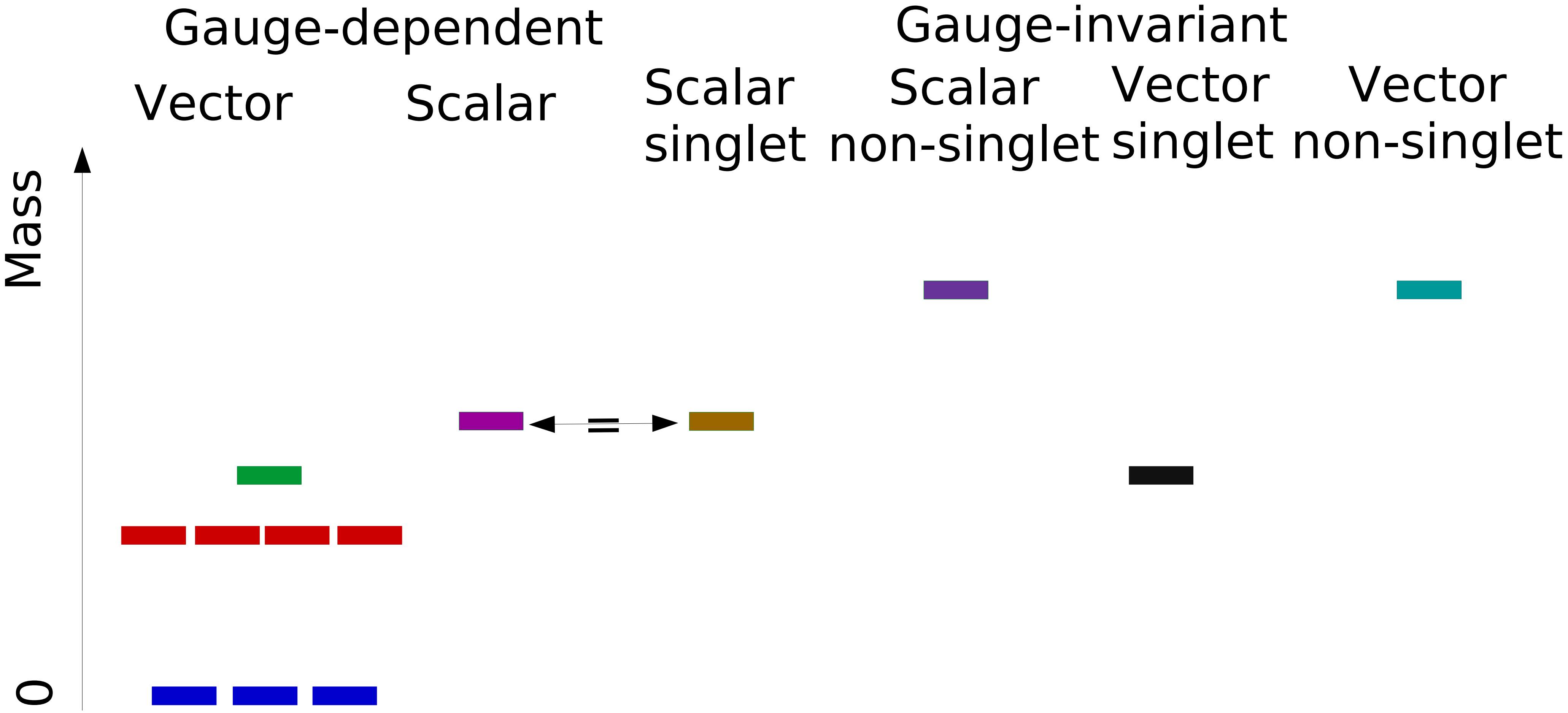

Gauge-dependent

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Vector

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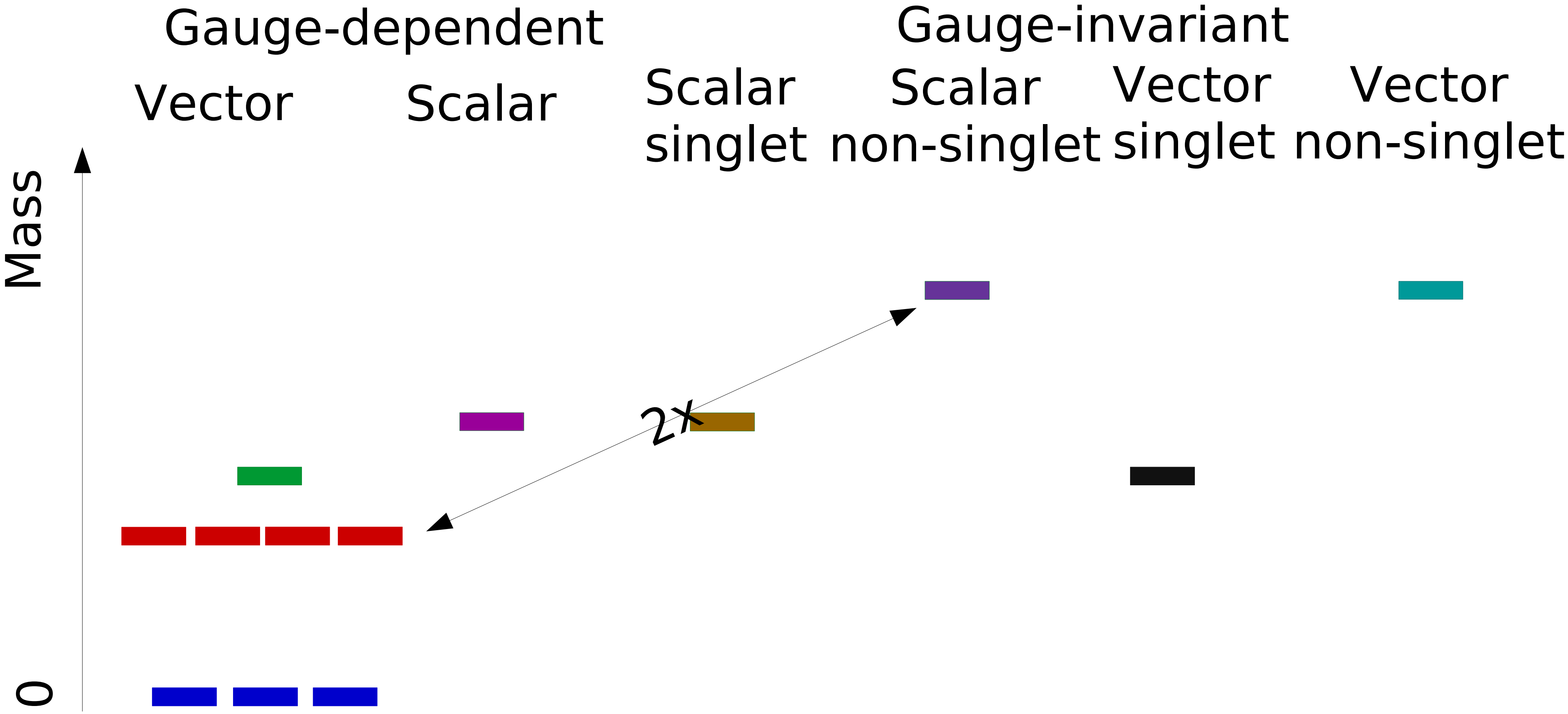

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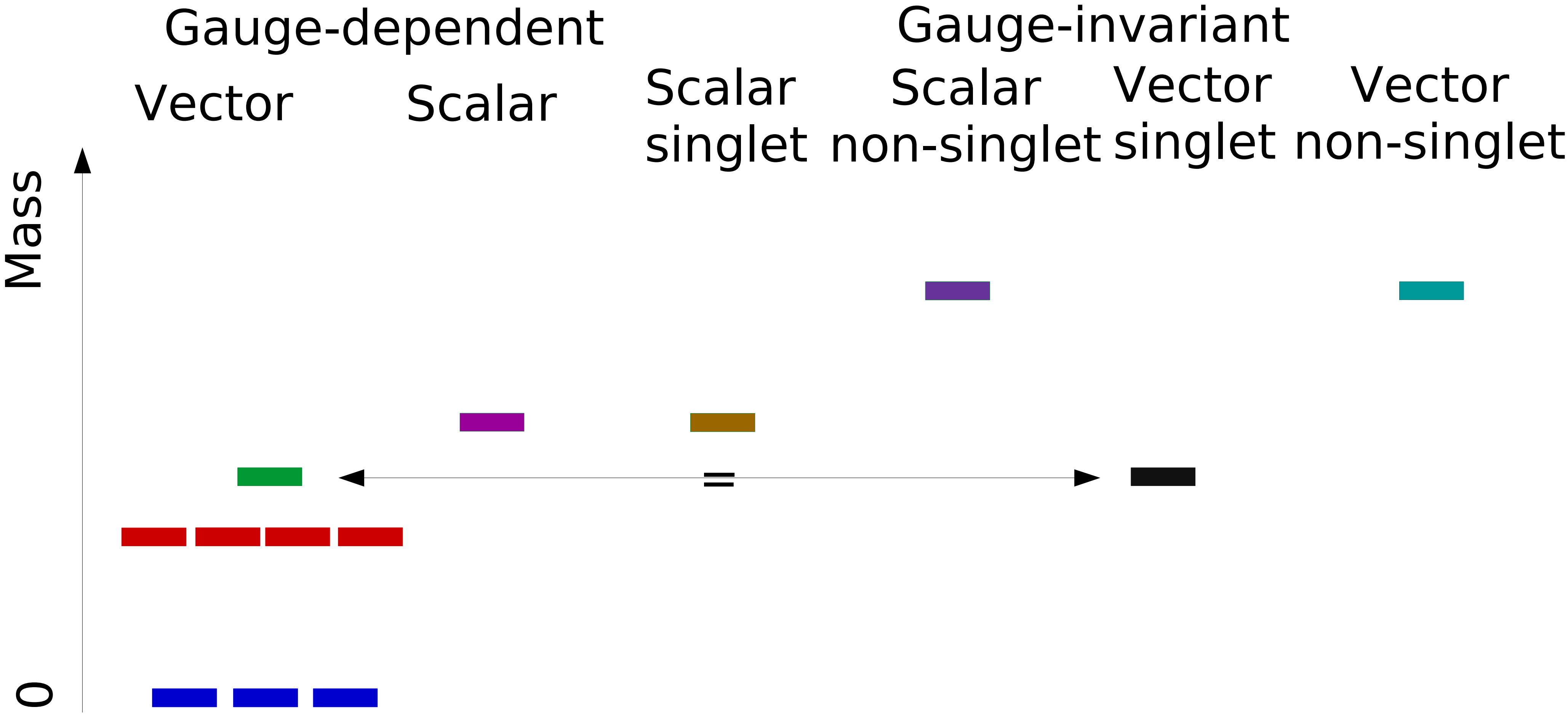

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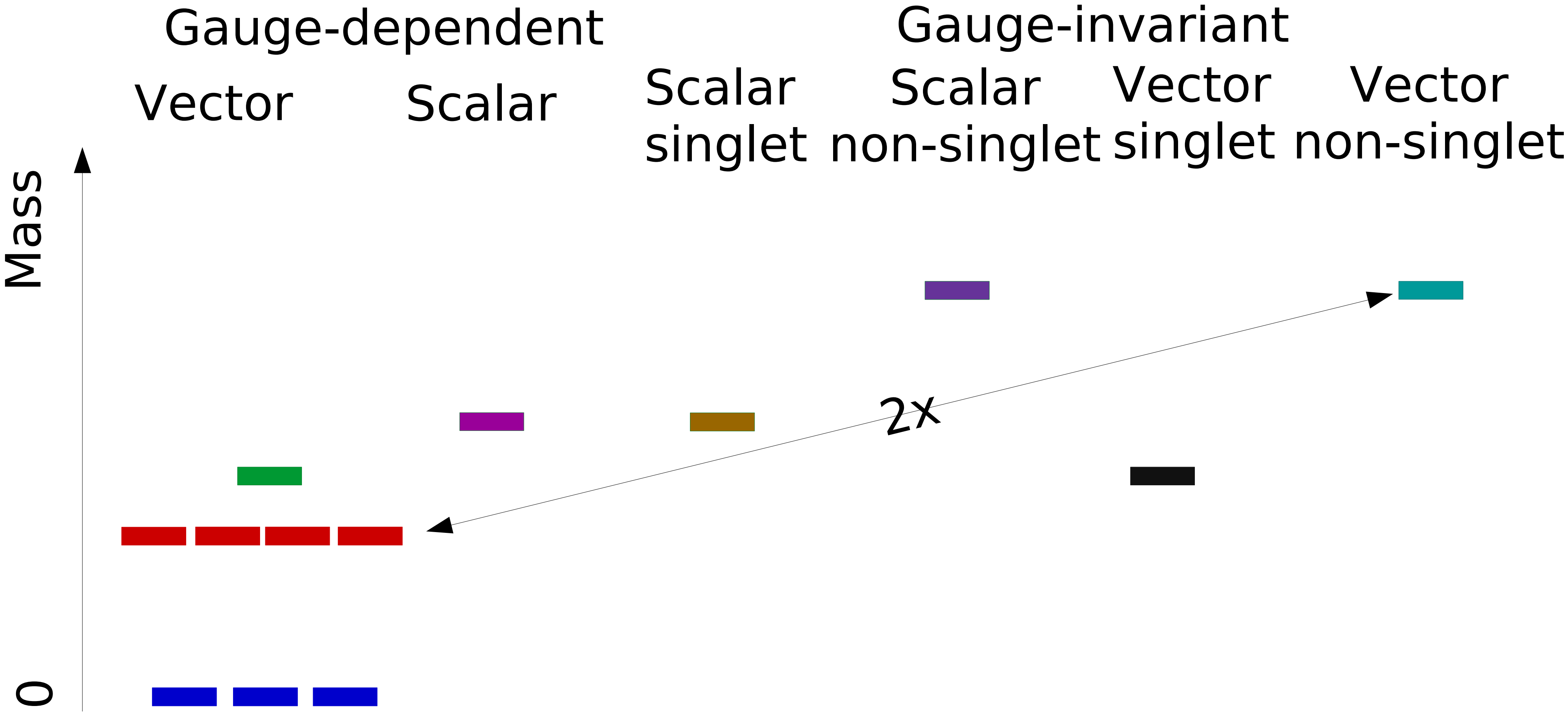

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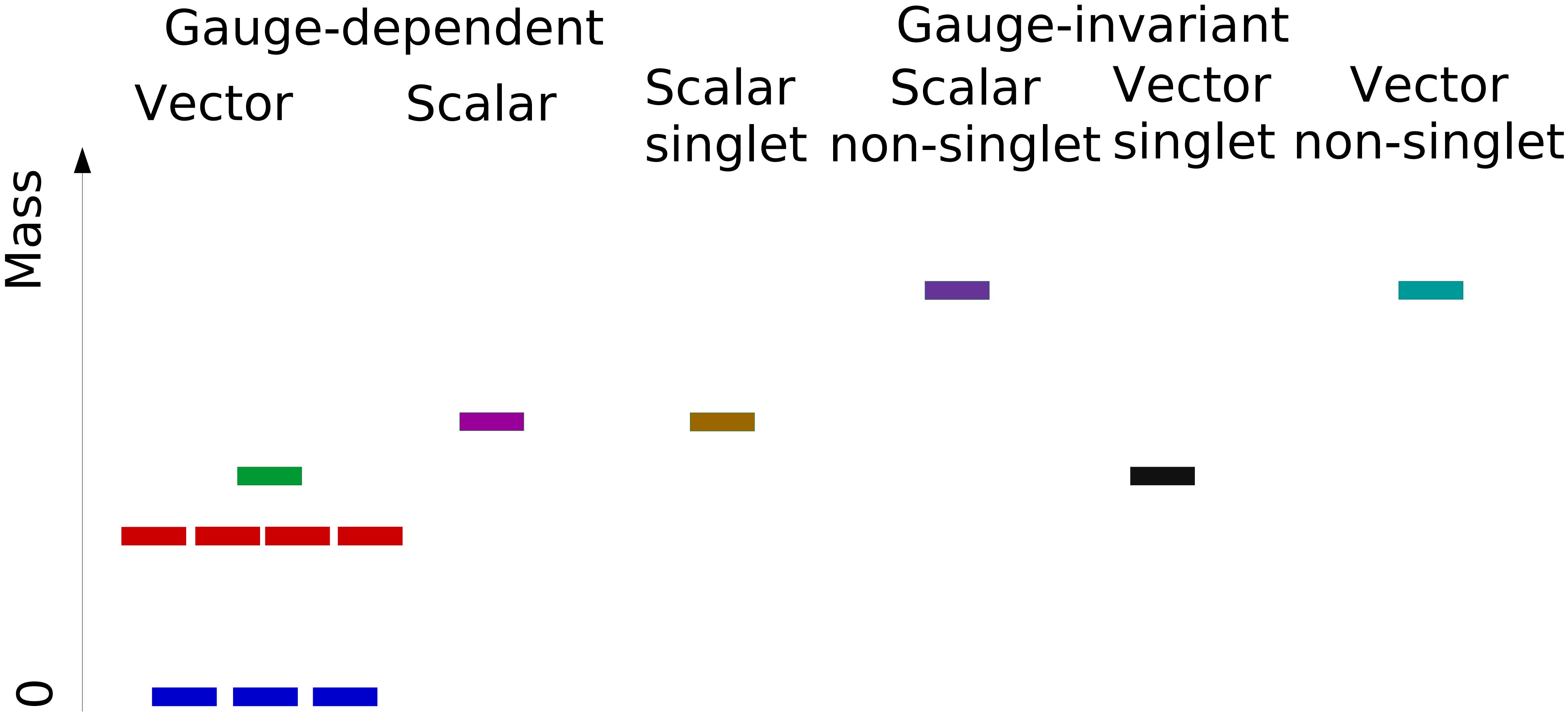

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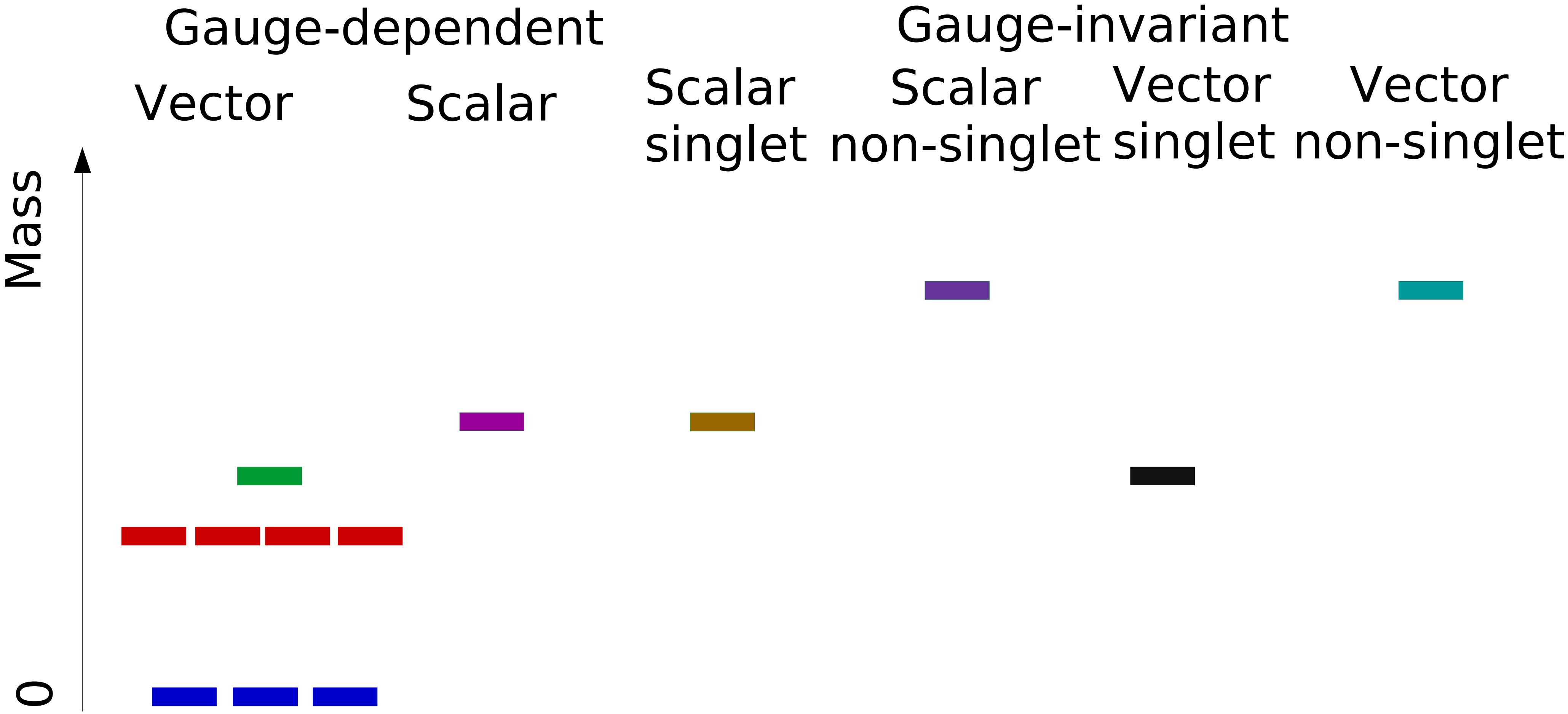

- Qualitatively different spectrum
- No mass gap! - But can be there: Adjoint Higgs

[Maas, Sondenheimer & Törek'17, Shigemitsu & Lee’85, Afferrante, Maas, Törek’19]

What about the vector?

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Matrix from group structure

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Only one state remains in the spectrum at mass of gauge boson 8 (heavy singlet)

Quantum gravity: Setting the scene

- QFT setting – no strings or other non-QFT settings

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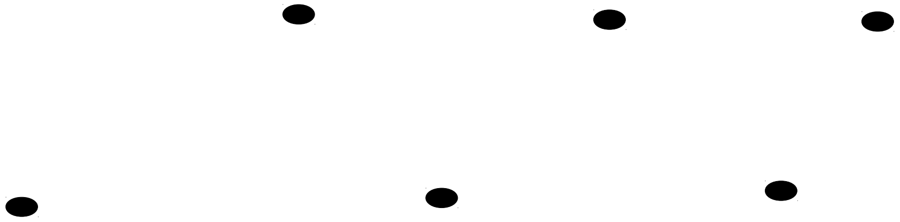
- Particle physics gauge symmetries and global symmetries should remain the same

Gravity as a gauge theory

[Hehl et al.’76]

Set of events with
neighbor relations



Gravity as a gauge theory

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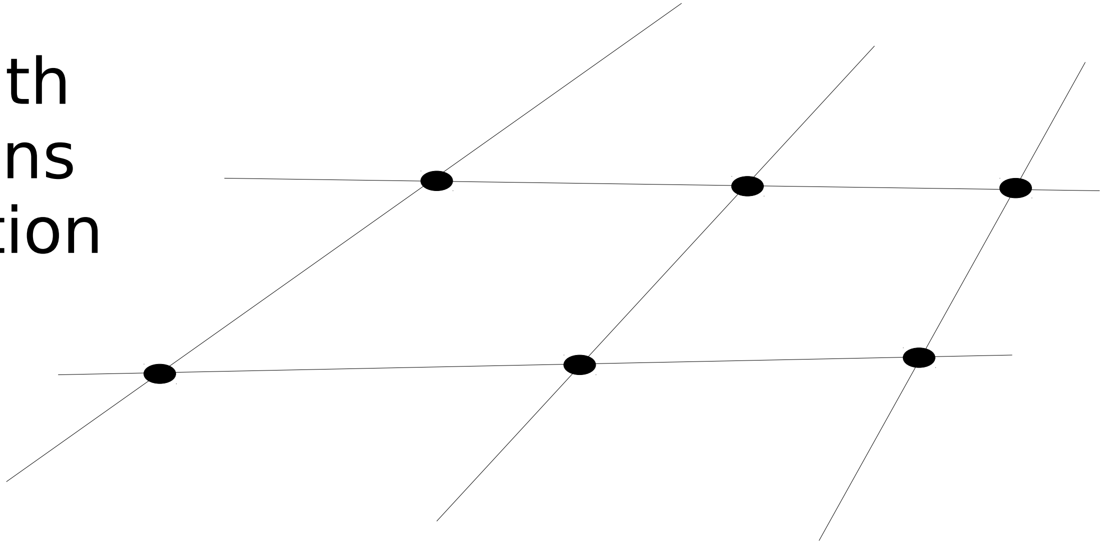
Set of events with
neighbor relations
E.g. R^4 in definition
of manifold



Gravity as a gauge theory

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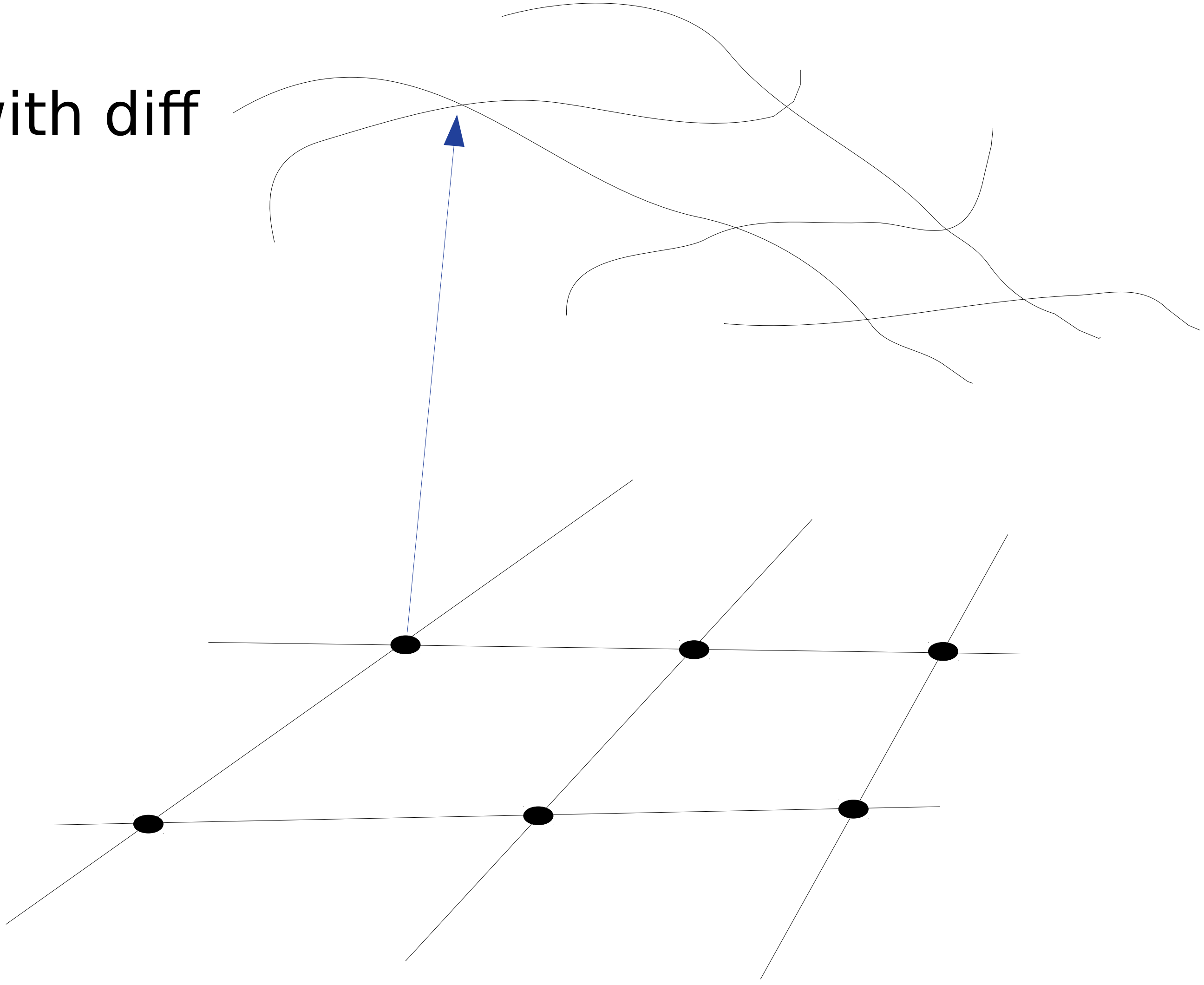
Manifold with diff
symmetry



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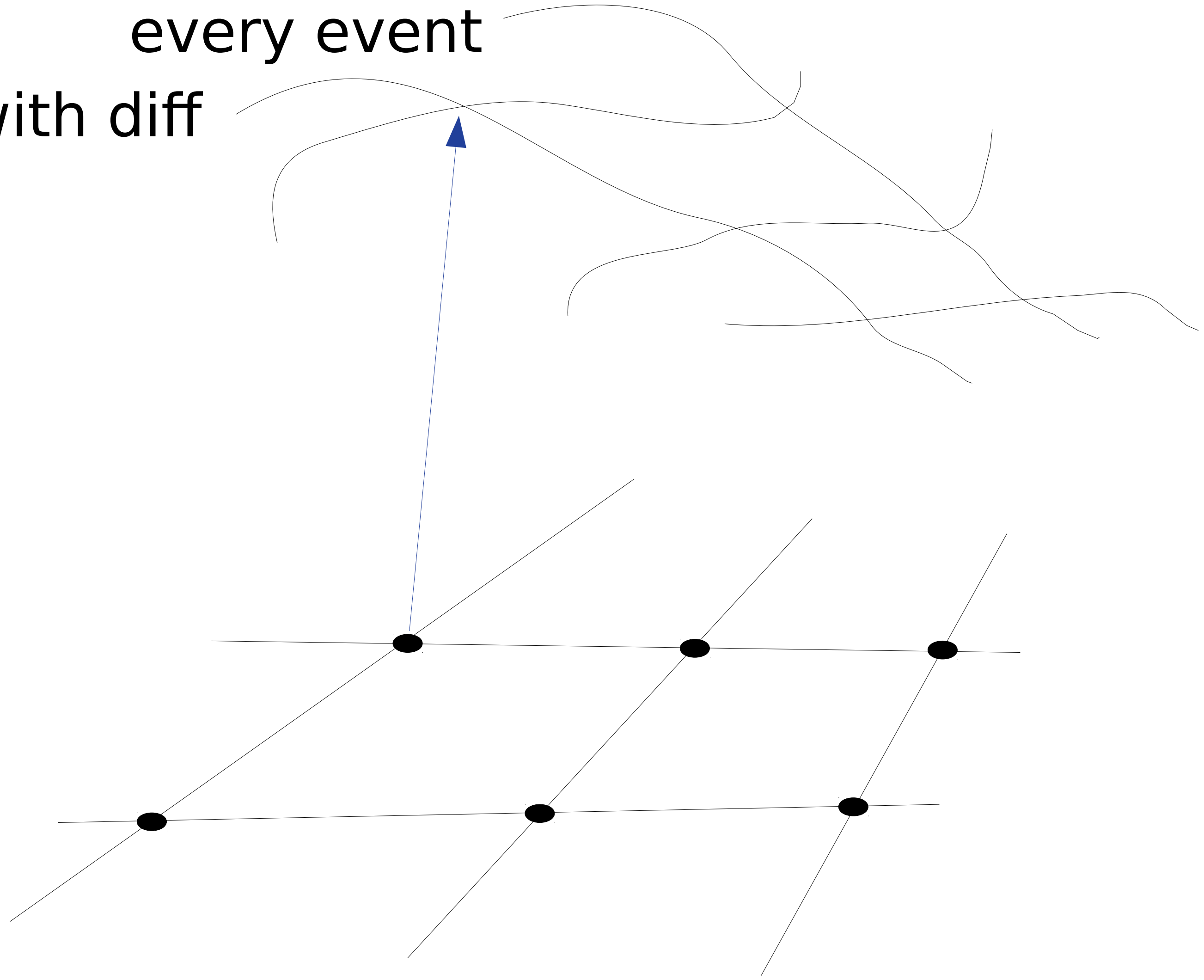

Metric g at every event

Manifold with diff symmetry

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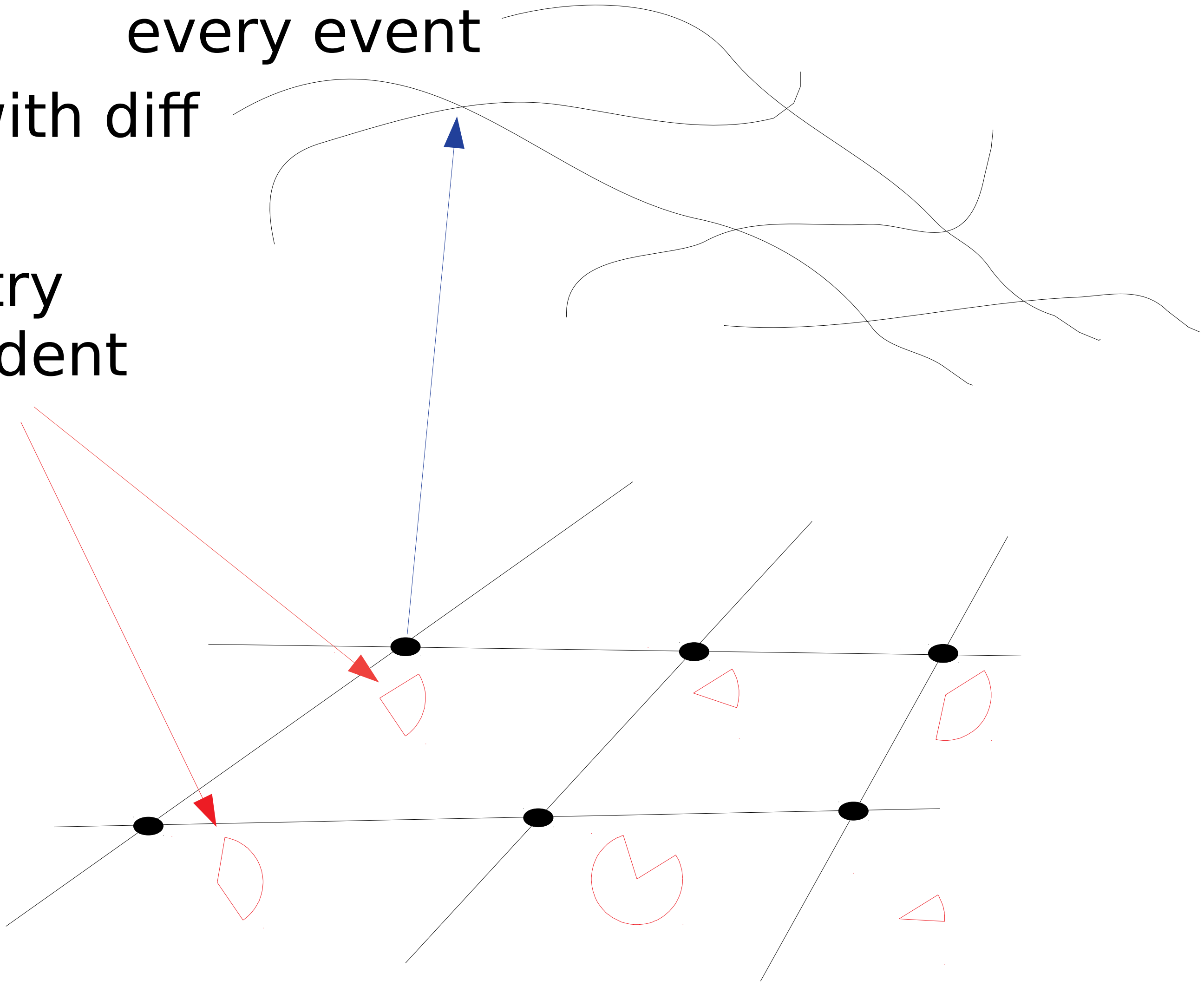

Metric g at
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Gauge symmetry
is event-dependent

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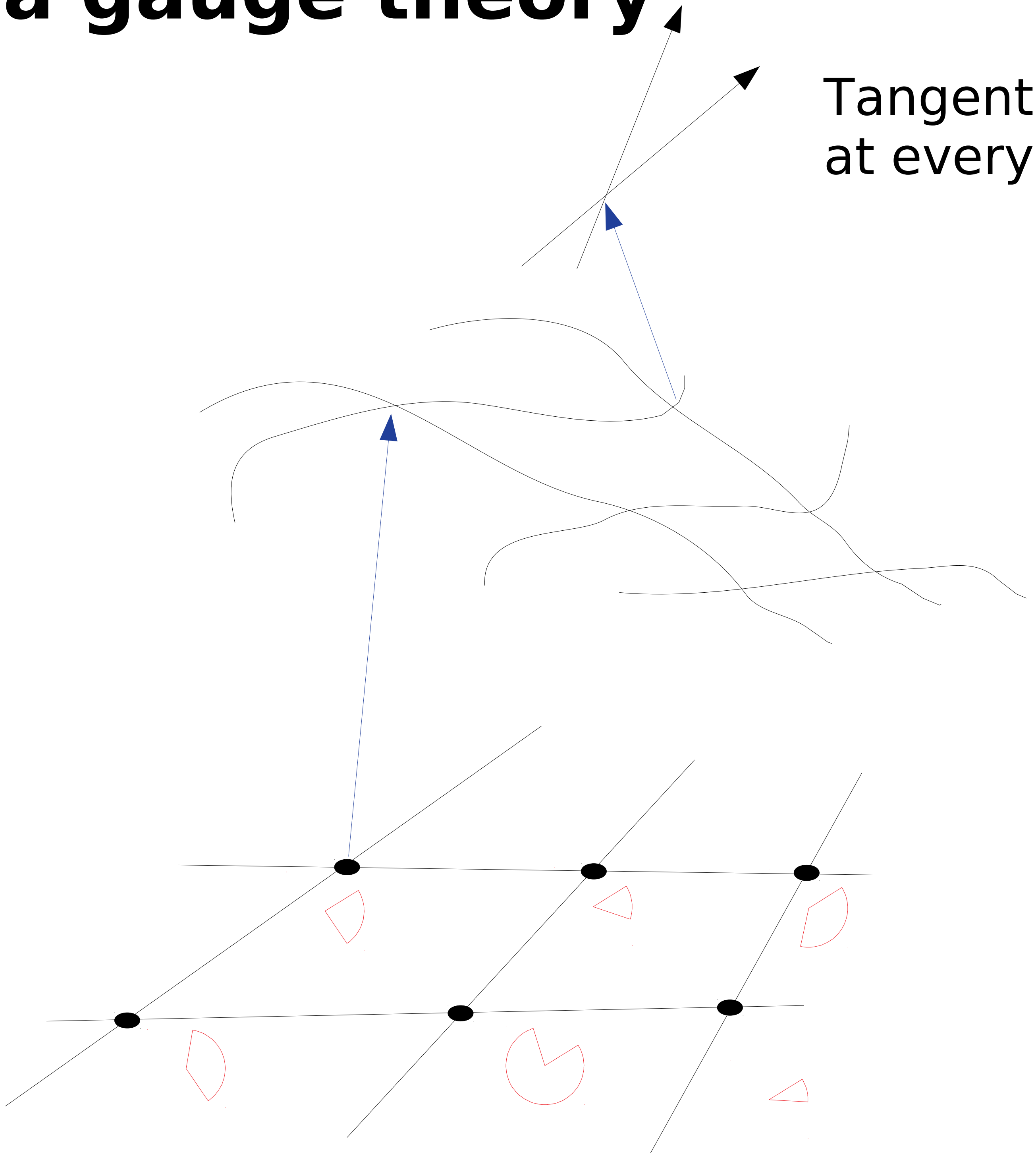

Tangent space
at every event

Gravity as a gauge theory

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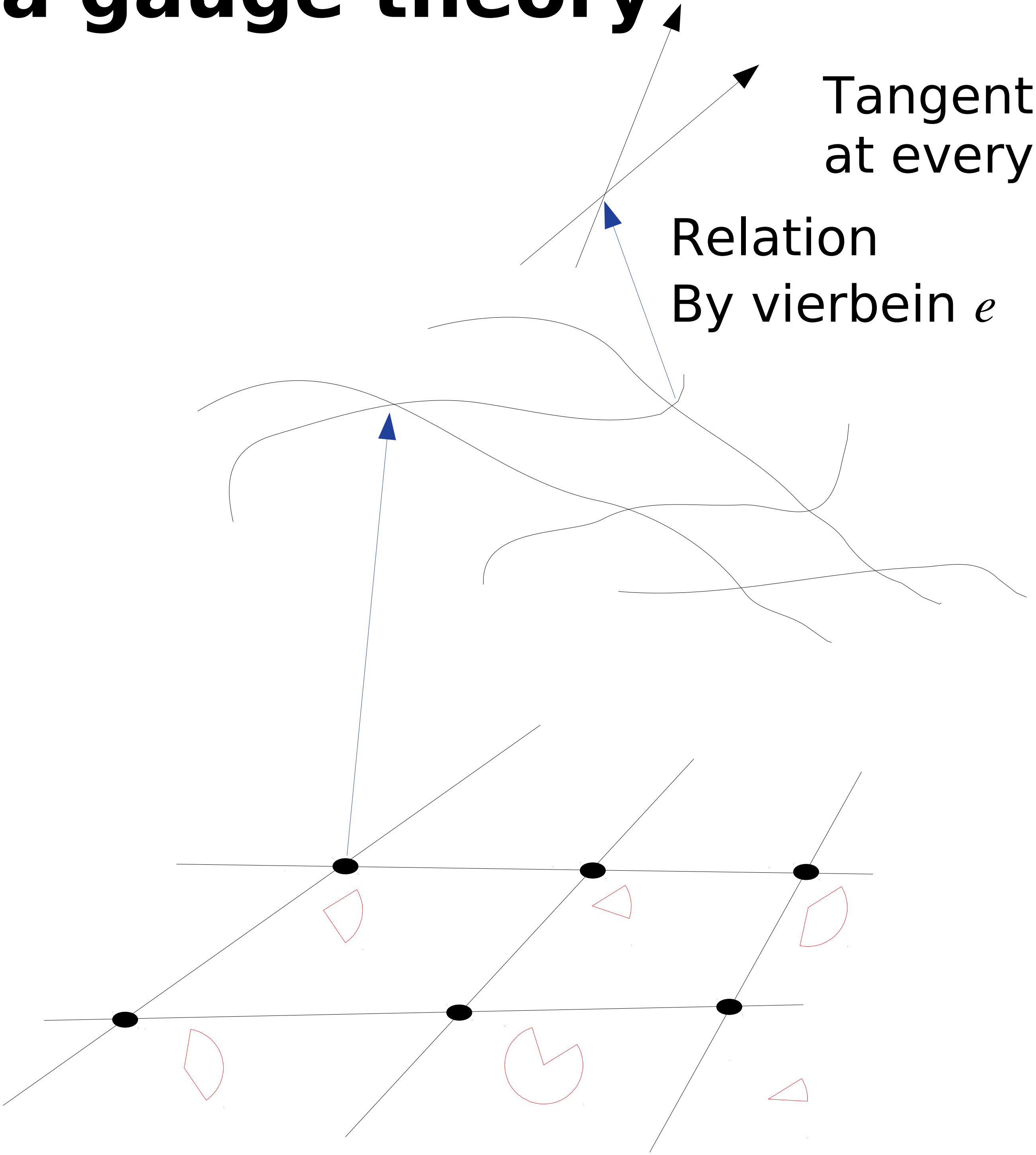

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Relation
By vierbein e

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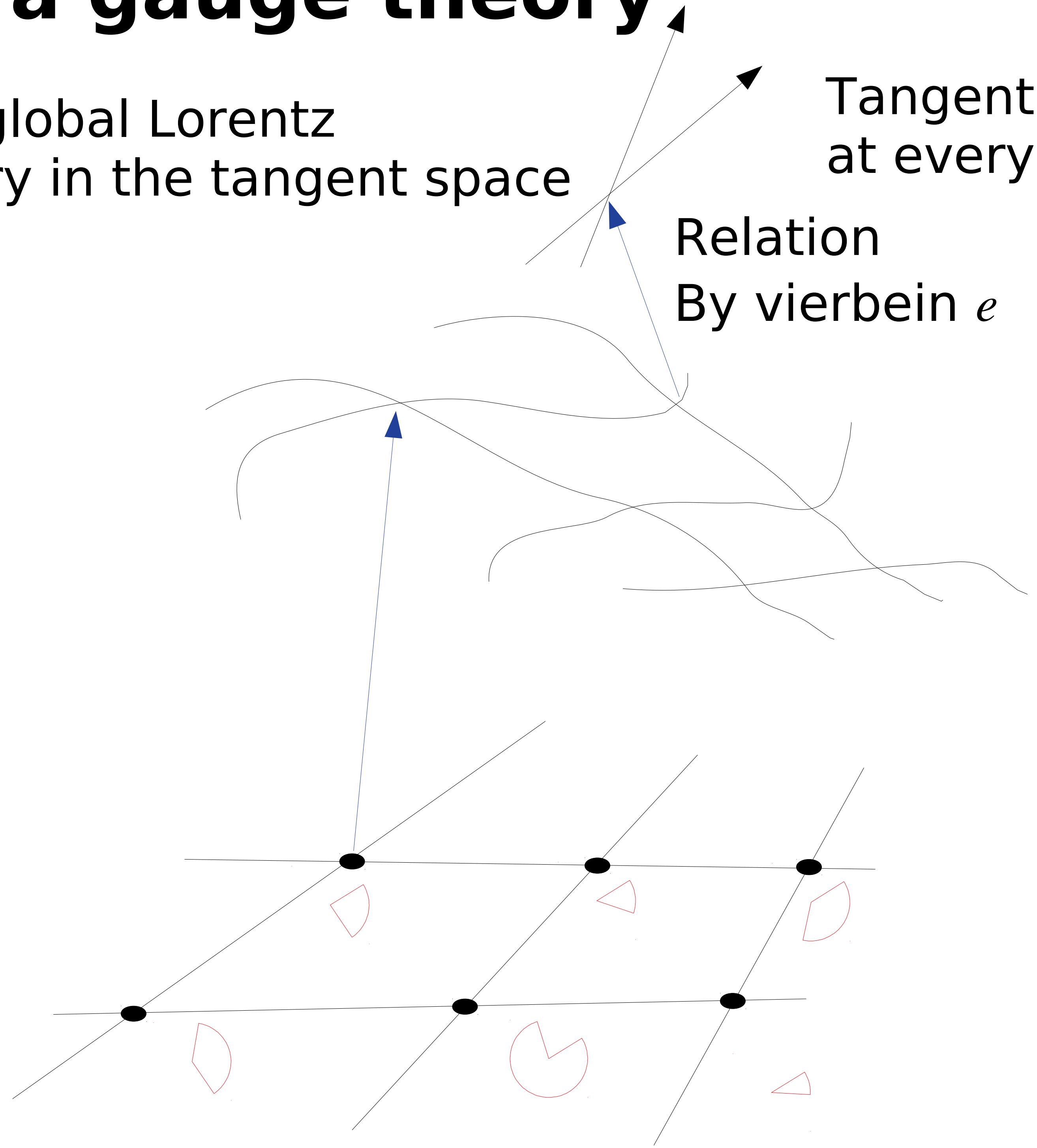

Spin as global Lorentz
symmetry in the tangent space

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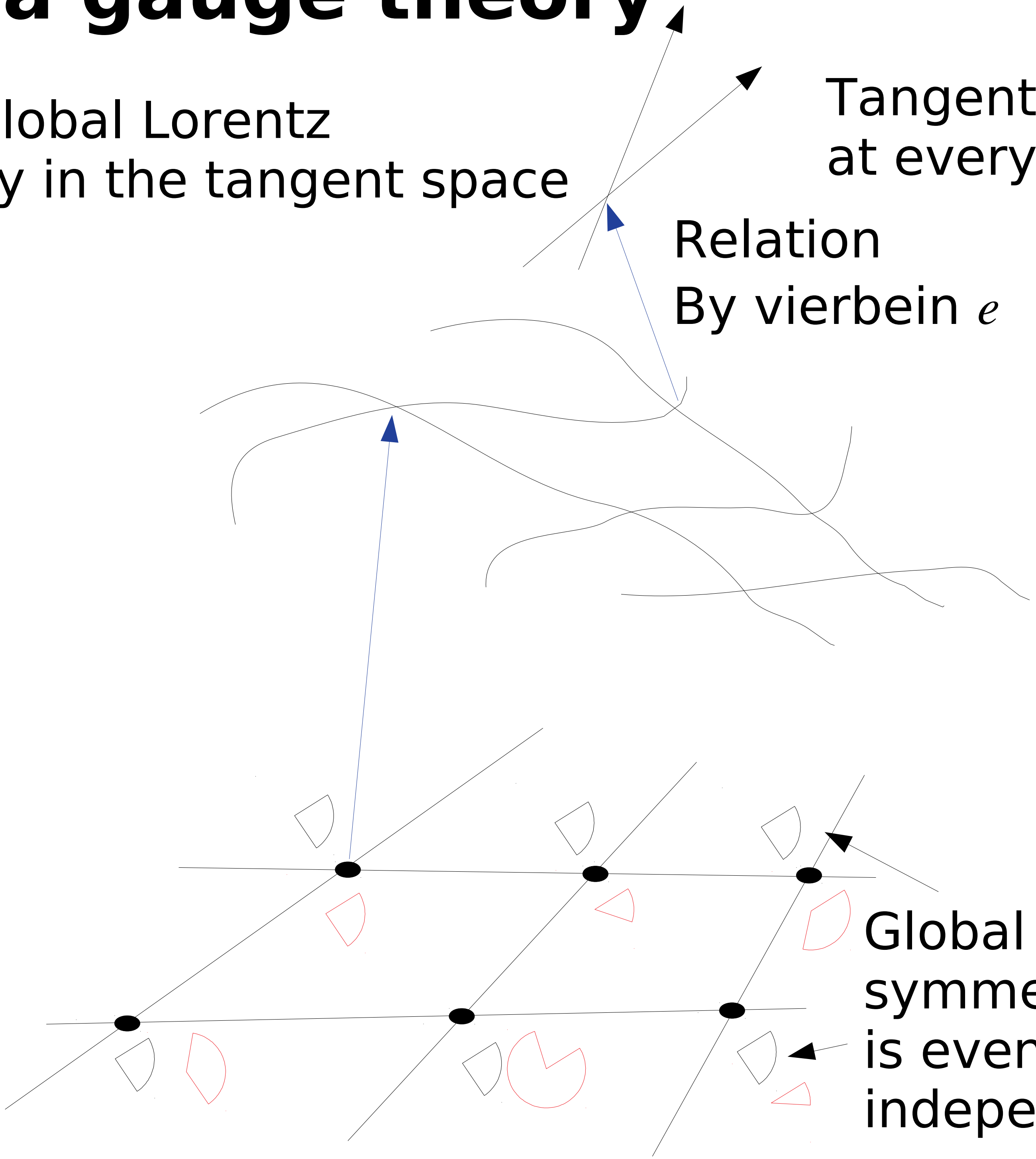

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Global
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Gravity as a gauge theory

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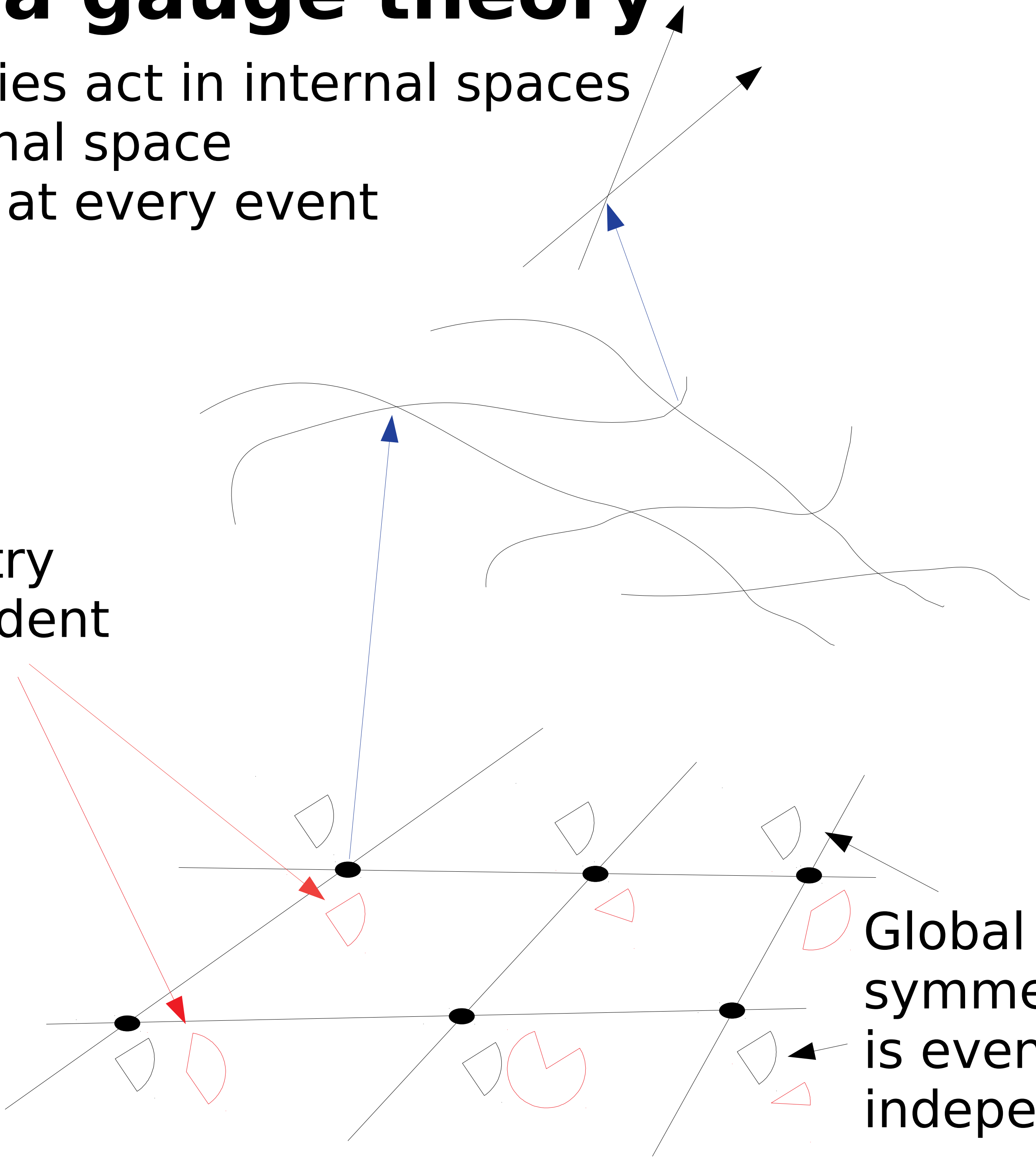
Internal symmetries act in internal spaces
Global: One internal space
Local: One space at every event



Gauge symmetry
is event-dependent

Global
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Dynamical formulation

$$Z = \int_{\Omega} D e_{\mu}^{a} D \phi^{a} e^{iS[\phi, e] + iS_{EH}[e]}$$

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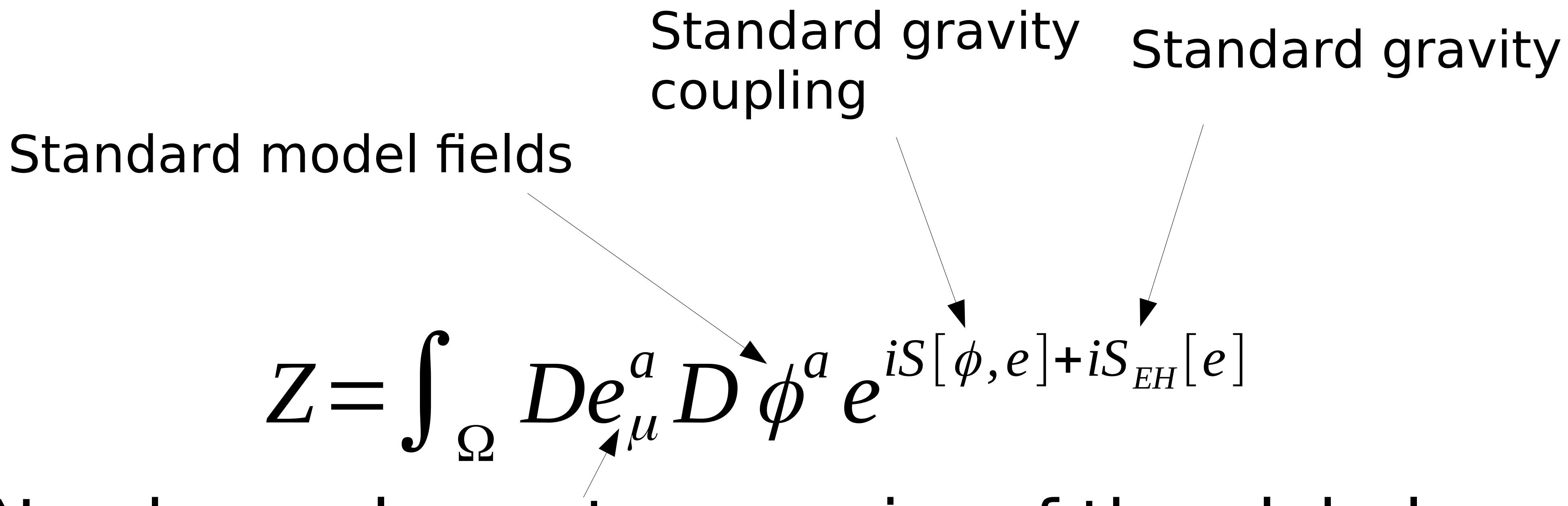
- Needs an elementary carrier of the global charge: Vierbein as dynamical degree of freedom
    - Other choices possibles

Dynamical formulation



Standard gravity coupling

Standard gravity

Standard model fields

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- Needs an elementary carrier of the global charge: Vierbein as dynamical degree of freedom
  - Other choices possibles
- Otherwise standard
  - E.g. Asymptotic safety for ultraviolet stability

Dynamical formulation

$$\langle O \rangle = \int_{\Omega} De_{\mu}^{a} D\phi^{a} O e^{iS[\phi,e]+iS_{EH}[e]}$$

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Needs to be invariant

- Locally under Diffeomorphism
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$$0 \neq \langle O \rangle = \int_\Omega De_\mu^a D\phi^a O e^{iS[\phi,e]+iS_{EH}[e]}$$

Needs to be invariant

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to be non-zero

Simpelst object: Scalar

- Consider a scalar particle

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Completely scalar: Invariant under all symmetries

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Argument is the event, not the coordinate

Result depends on events

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Simpelst object: Scalar

[Schaden'15]

$$\langle O(x) O(y) \rangle = D(r(x,y))$$

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Select geodesic

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[Schaden'15]

$$\langle O(x)O(y)\rangle = D(r(x,y))$$

Separate calculation

$$r(x,y)=\langle min_z \int_x^y d\lambda\, g_{\mu\nu}\frac{dz^\mu}{d\lambda}\frac{dz^\nu}{d\lambda}\rangle$$

- Distance is a quantum object: Expectation value
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  - Diff-invariant distance: Geodesic distance
  - Needs to be determined separately

Simpelst object: Scalar

[Schaden’15]

Reduces the full dependence: Definition
Dependence on events will only vanish if all events on the average are equal – probably true

$$\langle O(x)O(y)\rangle=D(r(x,y))$$

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- Generalization of flat-space arguments

Applying FMS

- Our universe is well-approximated by a classical metric
  - Due to the parameter values – special!
  - Small quantum fluctuations at large scales
    - Empirical result

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- Our universe is well-approximated by a classical metric
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  - Small quantum fluctuations at large scales
    - Empirical result
- FMS split after (convenient) gauge fixing
  - $g_{\mu\nu} = g^c_{\mu\nu} + \gamma_{\mu\nu}$
  - Classical part g^c is a metric, chosen to give exact (observed) curvature
  - Quantum part is assumed small

Distance

$$r(x,y)=\langle min_z \int_x^y d\lambda \, g_{\mu\nu} \frac{dz^\mu}{d\lambda} \frac{dz^\nu}{d\lambda} \rangle$$

- Application to the distance between two events

Distance

$$r(x,y)=\langle min_z \int_x^y d\lambda\, g_{\mu\nu} \frac{dz^\mu}{d\lambda} \frac{dz^\nu}{d\lambda} \rangle$$

$$=r^c(x,y)+\langle min_z \int_x^y d\lambda\, \gamma_{\mu\nu} \frac{dz^\mu}{d\lambda} \frac{dz^\nu}{d\lambda} \rangle = r^c+\delta r$$

Classical geodesic distance

- Application to the distance between two events
  - Yields to leading order classical distance

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Classical geodesic distance

Quantum corrections

- Application to the distance between two events
  - Yields to leading order classical distance
  - Size of quantum corrections depends on events

Propagators

$$\langle O(x) O(y) \rangle$$

Propagators

$$\langle O(x)O(y)\rangle = D_c(r^c) + \sum (\delta r)^n \partial_r^n D_c(r) + \langle O(x)O(y)\rangle_\gamma$$

$$D_c = \langle O(x)O(y)\rangle_{g^c}$$

- Double expansion

Propagators

$$\langle O(x)O(y)\rangle = D_c(r^c) + \sum (\delta r)^n \partial_r^n D_c(r) + \langle O(x)O(y)\rangle_\gamma$$

Leading term is
flat space propagator

$$D_c = \langle O(x)O(y)\rangle_{g^c}$$

- Double expansion

Propagators

Corrections from quantum
distance effects

$$\langle O(x)O(y)\rangle = D_c(r^c) + \sum (\delta r)^n \partial_r^n D_c(r) + \langle O(x)O(y)\rangle_\gamma$$

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- Double expansion
  - Quantum fluctuations in the argument

Propagators

Corrections from
metric fluctuations

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  - Quantum fluctuations in the action

Propagators

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- Double expansion
  - Quantum fluctuations in the argument
  - Quantum fluctuations in the action
- Reduces to QFT at vanishing gravity
  - Can be supplemented by FMS of BEH

Other operators

Other operators

Particles with spin – e.g. spin 1

$$e_{\mu}^{a}(x)O^{\mu}(x)$$

Other operators

Particles with spin – e.g. spin 1

Flat space operator

$$e_\mu^a(x)\,O^\mu(x)$$

Other operators

Particles with spin – e.g. spin 1

$$e_{\mu}^{a}\left(x\right)O^{\mu}\left(x\right)$$

Dressing for diff invariance

Other operators

Particles with spin – e.g. spin 1

Vector in tangent space

$$e_{\mu}^{a}\left(x\right)O^{\mu}\left(x\right)$$

Other operators

Particles with spin – e.g. spin 1

Vector in tangent space – like flavor and custodial charges

$$e_\mu^a\left(x\right)O^\mu\left(x\right)$$

Other operators

Particles with spin – e.g. spin 1

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Pure gravity excitations possible (Geons)

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Scalar: $R(x)$

Other operators

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Pure gravity excitations possible (Geons)

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Other operators

Particles with spin – e.g. spin 1

$$e_\mu^a(x)O^\mu(x)$$

Pure gravity excitations possible (Geons)

Graviton trace mode

Scalar: $R(x) \overset{e.g.\,flat,\,de\,Sitter,\,...}{=} const.+g_c^{\mu\nu}\gamma_{\mu\nu}+O(\gamma^2)$

Other operators

Particles with spin – e.g. spin 1

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Pure gravity excitations possible (Geons)

Graviton trace mode

Scalar: $R(x) \overset{e.g.flat, de\,Sitter, \ldots}{=} const.+g_c^{\mu\nu}\gamma_{\mu\nu}+O(\gamma^2)$

Massive? Stable? Dark matter?

Other operators

Particles with spin – e.g. spin 1

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Graviton

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  - Product of separate diff-invariant operators
    - Geon star: Similar to neutron star
    - Hawking radiation as tunneling

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- Simulations in quantum gravity?
    - Discretization on events as gauge theory?

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