Exact asymptotic safety: new solutions and large-N equivalences

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based on arXiv:1911.11168
in collaboration with Daniel Litim, Andrew Bond

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Outline

Motivation

- particle physics: search for extension / UV completion of the SM
  - ➔ common setup: perturbation theory, d=4, no gravity
  - ➔ asymptotic freedom as guiding principle

- progress in 2014: exact asymptotic safety in gauge-Yukawa theory
  [Litim, Sannino, JHEP 2014] [Litim, Sannino, Mojaza, JHEP 2016]
  - ➔ useful as template for model building
  - ➔ Are there more exact theories like this?

Recap: Asymptotic safety in 4d gauge theories

gauge theory with matter:

$$\alpha_g = \frac{g^2}{(4\pi)^2}$$

$$\beta_g = -B\,\alpha_g^2 + C\,\alpha_g^3 + \ldots$$

$B > 0:$ asymptotic freedom

$B \leq 0 \quad \Rightarrow \quad C > 0$ no UV fixed point

[Bond, Litim, EPJ 2017]

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gauge and Yukawa interaction:

$$\beta_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_y\alpha_g^2$$

$$\beta_y = \quad E\,\alpha_y^2 - F\,\alpha_g\alpha_y$$

$D,\ E,\ F \geq 0$

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$$D,\ E,\ F \geq 0 \qquad C' = C - \frac{DF}{E} < 0$$

$\hookrightarrow$ $B < 0,\ C' < 0:$ gauge-Yukawa UV fixed point

$$\alpha_y^* = \frac{F}{E}\alpha_g^* \qquad \alpha_g^* = B/C'$$

➔ Yukawa interaction is strictly required!

Example: Uncharged scalars

$$\beta_g\big|_{\beta_y=0} = -B\,\alpha_g^2 + \overbrace{\left[C - \frac{DF}{E}\right]}^{C'}\alpha_g^3 + \mathcal{O}\left(\alpha^4\right)$$

fermion & gauge boson interaction

uncharged scalars

Consider:

- non-abelian gauge
- charged fermions
- uncharged scalar

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Consider:

- non-abelian gauge
- charged fermions
- uncharged scalar

fermionic vs. scalar DOF

$B < 0$ $\quad$ $C' < 0$

What theories do actually allow for this UV fixed point?

Asymptotically safe template

[Litim, Sannino, JHEP 2014] [Litim, Sannino, Mojaza, JHEP 2016]

- $SU(N_c)$ gauge symmetry
- $SU(N_f) \times SU(N_f)$ global symmetry
- N_f quarks-like Dirac fermions $\Psi_i = (\Psi_{Li},\, \Psi_{Ri})^\intercal$
- $N_f \times N_f$ meson-like scalars H_{ij} (uncharged, complex matrix)

gauge coupling

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + \mathrm{Tr}\left(\overline{\Psi}\, i\not{D}\, \Psi\right) + \mathrm{Tr}\left(\partial_\mu H^\dagger \partial^\mu H\right)$$
$$- y\, \mathrm{Tr}\left(\overline{\Psi}_{Li} H_{ij} \Psi_{Rj} + \overline{\Psi}_{Ri} H^\dagger_{ij} \Psi_{Lj}\right) - u\, \mathrm{Tr}\left(H^\dagger H H^\dagger H\right) - v\left(\mathrm{Tr}\, H^\dagger H\right)^2$$

single Yukawa

two quartics

Exact perturbative control

- Veneziano limit: $N_{f,c} \to \infty$ but $N_f/N_c = \text{const.}$
- introduce 't Hooft couplings:

$$\alpha_g = \frac{d_R\, g^2}{(4\pi)^2} \qquad \alpha_y = \frac{d_R\, y^2}{(4\pi)^2} \qquad \alpha_u = \frac{N_f\, u}{(4\pi)^2} \qquad \alpha_v = \frac{N_f^2\, v}{(4\pi)^2}$$

- small expansion parameter: $\epsilon = \dfrac{N_f}{N_c} - \dfrac{11}{2}$ $\qquad -\dfrac{11}{2} < \epsilon < \infty$

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$$\begin{aligned}
\beta_g &= \alpha_g^2 \left[\boxed{\tfrac{4}{3}\,\epsilon} + \left(25 + \tfrac{26}{3}\epsilon\right)\alpha_g - 2\left(\tfrac{11}{2} + \epsilon\right)^2 \alpha_y\right] \\
\beta_y &= \alpha_y \left[(13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g\right] \\
\beta_u &= -\,(11 + 2\epsilon)\,\alpha_y^2 + 4\alpha_u\left(\alpha_y + 2\alpha_u\right) \\
\beta_v &= 12\alpha_u^2 + 4\alpha_v\left(\alpha_v + 4\alpha_u + \alpha_y\right)
\end{aligned}$$

1L gauge coefficient can be made arbitrarily small

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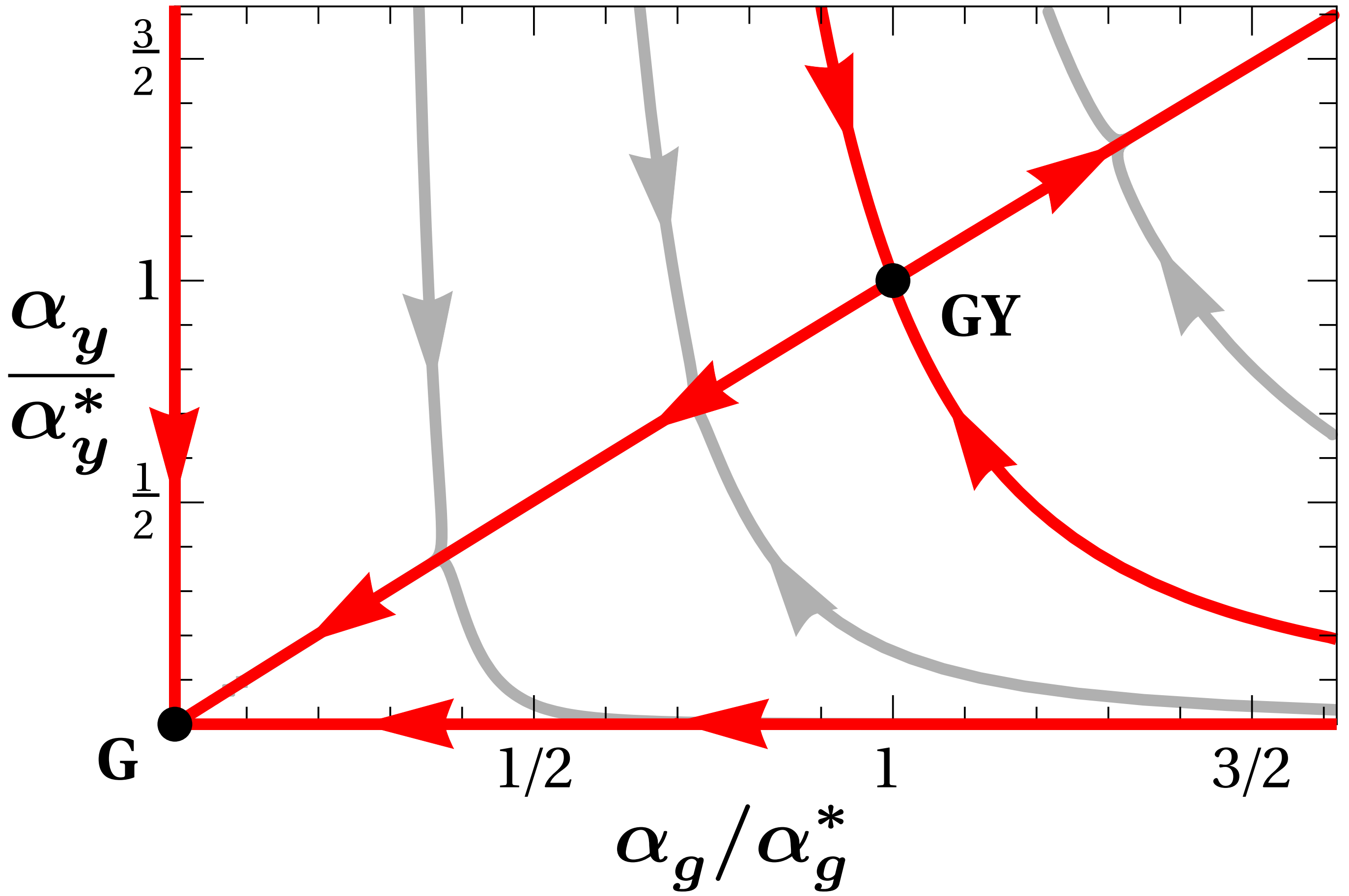
\end{aligned}$$

1L gauge coefficient can be made arbitrarily small

- ➔ fixed point couplings, critical exponents under perturbative control for $|\epsilon| \ll 1$
- ➔ higher loop orders suppressed with higher powers of expansion parameter

Phase diagram SU(N)

$0 < \epsilon \ll 1$



UV conformal window

- $-\frac{11}{2} < \epsilon < 0$ asymptotic freedom

  $0 < \epsilon < \epsilon_{\max}$ asymptotic safety

- UV conformal window studied at highest available loop orders

  [Bond, Litim, Medina, TS, PRD 2018]

  - ➔ 3-loop in gauge, 2-loop in Yukawa, quartic RGEs
  - ➔ NLO in ϵ-expansion
  - ➔ $\epsilon_{\max} \;\Rightarrow\; N_c^{\min} = 5\,..\,7$

Is this template model unique? Are there more 4d theories with exact asymptotic safety?

Alternative exact theories ?

- remember, we want $N_{c,f} \to \infty$ and $N_f / N_c = \text{const.}$

$$\beta_g = -\frac{4}{3}\left[\,\frac{11}{2} C_2^G - (2N_f)\, S_2^R\,\right] \frac{\alpha_g^2}{d_R} = -\frac{4}{3}\epsilon\, \alpha_g^2$$

gauge bosons

fermions

$$SU(N_c): \;\; C_2^G = N_c$$

R	d_R	S_2^R
fund.	N_c	$1/2$
antisymm.	$N_c\,(N_c - 1)\,/2$	$N_c/2 - 1$
symm.	$N_c\,(N_c + 1)\,/2$	$N_c/2 + 1$
adj.	$N_c^2 - 1$	N_c

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gauge bosons   fermions

$SU(N_c):\ C_2^G = N_c$

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symm.	$N_c(N_c+1)/2$	$N_c/2+1$
adj.	N_c^2-1	N_c

- how about other non-abelian gauge groups?
  - exceptional ones do not have N_c
  - orthogonal or symplectic are the only options

$SO(N_c):\ C_2^G = N_c/2-1$

R	d_R	S_2^R
fund.	N_c	$1/2$
adj.	$N_c(N_c-1)/2$	$N_c/2-1$
symm.	$(N_c/2+1)(N_c-1)/2$	$N_c/2+1$

$Sp(N_c):\ C_2^G = N_c/2+1$

R	d_R	S_2^R
fund.	N_c	$1/2$
antisymm.	$(N_c/2-1)(N_c+1)$	$N_c/2-1$
adj.	$N_c(N_c+1)/2$	$N_c/2+1$

Template model with SO, Sp

- universal definition $\epsilon = \dfrac{N_f}{N_c} - \dfrac{11}{2}$ for $SU\,(N_c)\,,\ SO\,(2N_c)\,,\ Sp\,(2N_c)$
- RGEs imply

$$\alpha_i^{SO} = \alpha_i^{Sp}$$

$$\beta_g = \alpha_g^2 \left[\tfrac{2}{3}\,\epsilon + \left(\tfrac{25}{4} + \tfrac{13}{6}\epsilon\right)\alpha_g - \tfrac{1}{2}\left(\tfrac{11}{2} + \epsilon\right)^2 \alpha_y\right]$$

$$\beta_y = \alpha_y\left[\left(\tfrac{15}{2} + \epsilon\right)\alpha_y - 3\,\alpha_g\right]$$

$$\beta_u = -\left(\tfrac{11}{2} + \epsilon\right)\alpha_y^2 + 4\alpha_u\left(\alpha_y + 2\alpha_u\right)$$

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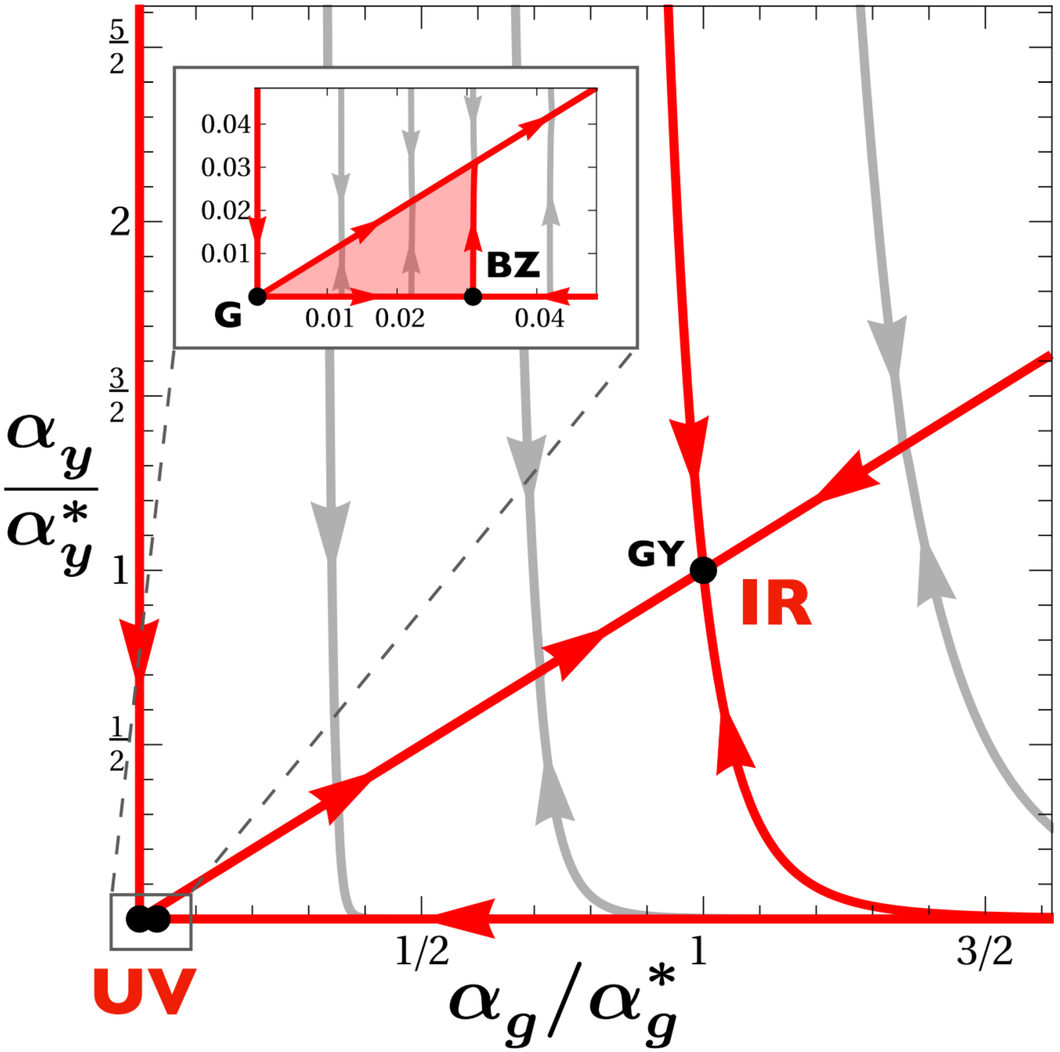
- fixed point requires $B,\ C' < 0,\ B/C' \ll 1$, but:

$$\frac{C'}{C}\Big|_{B\to -0} \approx \begin{cases} -0.117 & \text{for} \quad SU\left(N_c\right) \\ +0.032 & \text{for} \quad SO\left(2N_c\right),\ Sp\left(2N_c\right) \end{cases}$$

➔ gauge-Yukawa fixed point is only UV in the SU(N) case

Phase diagram SO(2N), Sp(2N)



$$-1 \ll \epsilon < 0$$

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- Why? Ratio #scalars / #fermions:

$$N_f \approx \frac{11}{2} N_c$$

Gauge group	Weyl fermions	Real scalars
$SU(N_c)$	$2N_f N_c$	$2N_f^2$
$SO(2N_c)\,,\ Sp(2N_c)$	$4N_f N_c$	$2N_f^2$

➔ Is SU(N) special, or can we reduce the number of fermions?

The Majorana cure

- introduce Majoranas $\Psi^c = \Psi$ to reduce fermion DOF

$$\Psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} \quad \Rightarrow \quad \frac{1}{\sqrt{2}} \begin{pmatrix} \psi \\ \psi^c \end{pmatrix}$$

This is **forbidden** for $SU(N_c)$ due to chiral anomalies!

$SO(N_c)$: fundamental rep. is **real** $(t^a)^\intercal = -t^a$

$Sp(N_c)$: fundamental rep. is **pseudoreal** $(t^a)^\intercal = -M t^a M^{-1}$

$\longrightarrow$ Anomalies cancel: $d^{abc} = \frac{1}{2}\mathrm{Tr}\left[t^a \left\{t^b, t^c\right\}\right] = 0$

Majoranas in action

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \mathrm{Tr}\left(\psi^\dagger i\sigma^\mu D_\mu\,\psi\right) + \mathrm{Tr}\left(\partial_\mu H^\dagger \partial^\mu H\right) - u\,\mathrm{Tr}\,(H^\dagger H)^2 - v\,(\mathrm{Tr}\,H^\dagger H)^2 + \ldots$$

$SO\,(N_c)$

$$\ldots - \tfrac{1}{2}\,y\,\mathrm{Tr}\left(\psi^\intercal h\,H\varepsilon\,\psi + \psi^\dagger h\,H^\dagger\varepsilon\,\psi^*\right)$$

symmetric gauge contraction

$$H_{ij} \Rightarrow H_{(ij)}$$

$Sp\,(N_c)$

$$\ldots - \tfrac{1}{2}\,y\,\mathrm{Tr}\left(\psi^\intercal f\,H\varepsilon\,\psi + \psi^\dagger f\,H^\dagger\varepsilon\,\psi^*\right)$$

antisymmetric gauge contraction

$$H_{ij} \Rightarrow H_{[ij]}$$

In both cases, the flavour symmetry is $SU\,(N_f)$

Majorana RGEs

$$SO(N_c)\,,\;\; Sp(N_c):$$

$$\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

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\end{aligned}$$

$$\frac{C'}{C}\Big|_{B\to -0} \approx -0.117$$

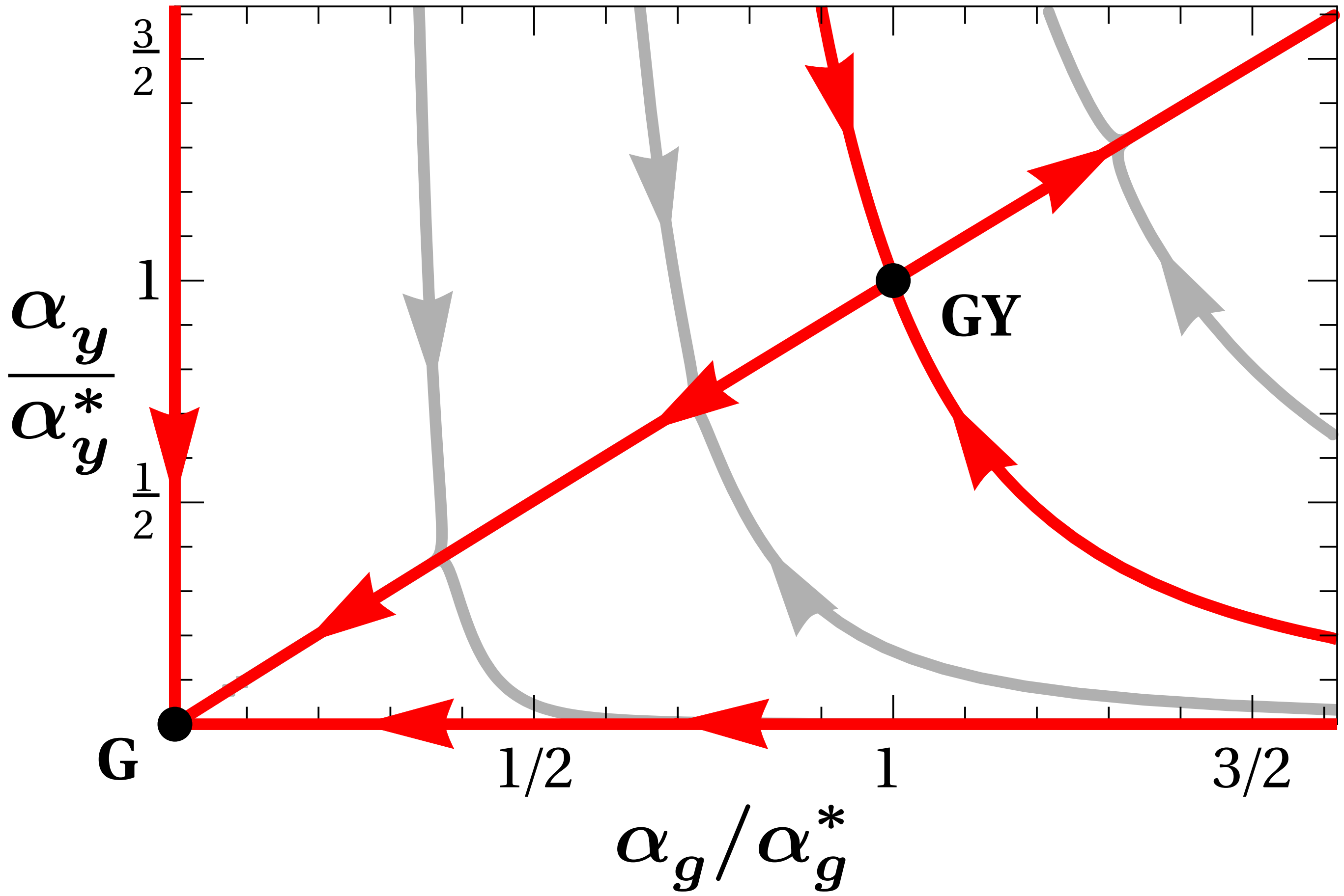
(+ subl.) for all 3 cases: $SO(N_c)\,,\; Sp(N_c)$ with Majorana fermions

$SU(N_c)$ with Dirac fermions

➔ exact asymptotic safety discovered in SO(N), Sp(N) theories!

Phase diagram SO(N), Sp(N)

$0 < \epsilon \ll 1$



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$SU(N_c)$ with Dirac fermions

Even away from the fixed point: $\boxed{\alpha_i^{SO} = \alpha_i^{Sp} = 2\,\alpha_i^{SU}}$

How can this equivalence be understood?

$SO(N_c)$ gauge group
$SU(N_f)$ global symmetry
Majorana fermions, $H_{(ij)}$

$Sp(N_c)$ gauge group
$SU(N_f)$ global symmetry
Majorana fermions, $H_{[ij]}$

$$2\,\alpha_i^{SU} = \alpha_i^{SO} = \alpha_i^{Sp}$$

$SU(N_c)$ gauge group
$SU(N_f) \times SU(N_f)$ global
Dirac fermions, H_{ij}

$SO(2N_c)$ gauge group
$SU(N_f) \times SU(N_f)$ global
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negative
dimensionality

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orbifold
reduction

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Negative dimensionality theorems

- at finite n, m:

$$SU(n) = \overline{SU}(-n), \quad SO(m) = \overline{Sp}(-m)$$

  - ➔ exchange symmetr. $\leftrightarrow$ antisymmetr.

[King, J.Math. Phys. 12, 1588 (1971)]
[Mkrtchian, Phys. Lett. 105B, 174 (1981)]
[Cvitanovic, Kennedy, Phys. Scripta 26, 5 (1982)]

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- apply both to gauge- and flavour group: „theory“ with $-N_c,\ -N_f$
- can map back $-N_c \mapsto +N_c,\ -N_f \mapsto +N_f$ in the large-N limit
  - $\alpha_i \mapsto \alpha_i$ ‘t Hooft couplings are matched
  - $\epsilon \mapsto \epsilon$ ratios N_f/N_c do not change
  - corrections $\mathcal{O}\left(N^{-1}\right)$ are subleading

$$\underbrace{\beta_i\big|_{SO(N_c)\times SU(N_f)} = \beta_i\big|_{\overline{Sp}(-N_c)\times\overline{SU}(-N_f)} \overset{N_{c,f}\to\infty}{=} \beta_i\big|_{\overline{Sp}(N_c)\times\overline{SU}(N_f)}}_{\text{RGEs of ‘t Hooft couplings}}$$

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Negative dimensionality theorems

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$\longleftrightarrow$

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Orbifold reduction – a recipe

- identify parent theory
- identify discrete subgroup of global symmetry, eliminate all DOF not invariant
  - ➔ generates projection $\mathcal{P}$
- $\mathcal{P}: \text{ parent theory} \mapsto \text{child theory}$ for classical fields, operators
- for certain choices:

$$\mathcal{P}\,\langle\text{neutral single trace operator}\rangle\Big|_{\text{parent}} \sim \langle\text{neutral single trace operator}\rangle\Big|_{\text{child}}$$

  - ➔ this hold to in all loop orders at large-N

[Bershadsky, Johansen, Nucl. Phys. B536, 141 (1998)]
[Schamltz, Phys. Rev. D59, 105018 (1999)]
[Erlich, Naqvi, JHEP 12 ,047 (2002)]
[Armoni, Shifman, Veneziano, Nucl. Phys. B667, 170 (2003)]
[Kovtun, Unsal, Yaffe, JHEP 07, 008 (2005)]

Orbifold reduction

parent

$SO(2N_c)$ gauge group
$SU(2N_f)$ global symmetry
Majorana fermions, $H_{(ij)}$

parent

$Sp(2N_c)$ gauge group
$SU(2N_f)$ global symmetry
Majorana fermions, $H_{[ij]}$

child

$SU(N_c)$ gauge group
$SU(N_f) \times SU(N_f)$ global
Dirac fermions, H_{ij}

gauge bosons	Weyl fermions	Real scalars
$2N_c^2$	$4N_cN_f$	$4N_f^2$
N_c^2	$2N_cN_f$	$2N_f^2$

➔ child theory has only half the DOF

$$\alpha_i^{SO} = \alpha_i^{Sp} = \boxed{2}\alpha_i^{SU}$$

[Hanada, Yamamoto, JHEP 02, 138 (2012)]

Conclusion

- exact asymptotic safety with SU, $\underbrace{\text{SO and Sp}}_{\text{requires Majoranas}}$ gauge groups

  requires Dirac fermions

- large-N equivalences due to negative dimensionality theorems and orbifold reduction
  - ➔ triality of asymptotic safe theories
  - ➔ duality of IR conformal theories
  - ➔ many more equivalences in reach