Spectroscopy of the Reuter Fixed Point from the composite operator formalism

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IN·DEI·NOMINE·FELICITER

Netherlands Organisation for Scientific Research

W. Houthoff, A. Kurov, F.S., arXiv:2002.00256

A. Kurov, F.S., arXiv:2003.07454

International Asymptotic Safety Seminar, April 6$^{\rm th}$, 2020

Outline

- Asymptotic Safety and the Functional Renormalization Group
- composite operator formalism
- results I: scaling of geometric operators
- results II: spectroscopy of the Reuter fixed point
- concluding thoughts

Asymptotic Safety in a nutshell

Asymptotic Safety: the arena

asymptotic safety = effective field theory + predictive power

- theory $\Leftarrow$ specify
  - a) field content (e.g. graviton)
  - b) symmetries (e.g. coordinate transformations)

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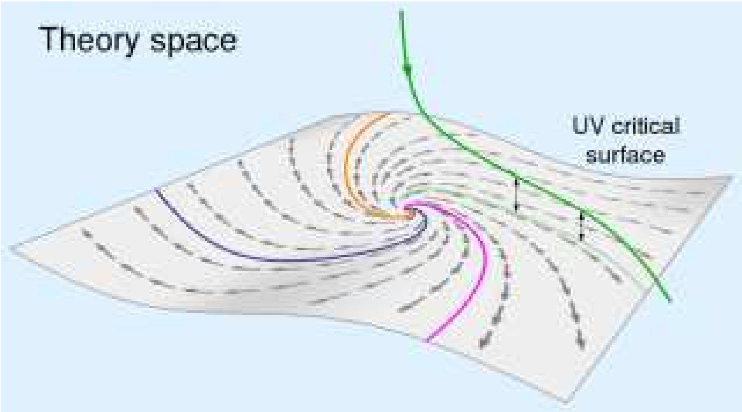
- Wilsonian renormalization group flow:
  - couplings run when integrating out quantum fluctuations at scale k

    (e.g. $k\partial_k g_i = \beta_i(g_i)$)

UV-completion of a quantum field theory

high energy behavior controlled by renormalization group fixed point



[scholarpedia ’13]

- 2 classes of renormalization group trajectories:
  - relevant = end at fixed point in UV
  - irrelevant = go somewhere else...
- theory ending at the fixed point is free of unphysical UV divergences
- predictive power:
  - number of relevant directions $\Longleftrightarrow$ free parameters (experimental input)

asymptotic freedom and asymptotic safety

Renormalization via UV fixed points $\Longrightarrow$ two classes of renormalizable QFTs

- Gaussian Fixed Point (asymptotic freedom)
  - fundamental theory: free
  - critical exponents: classical power counting
- non-Gaussian Fixed Point (asymptotic safety)
  - fundamental theory: interacting
  - critical exponents: classical power counting + quantum corrections

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gravitational asymptotic safety program

gravity in $d = 4$ is controlled by NGFP (Reuter fixed point)

The Asymptotic Safety package

Reuter fixed point (NGFP)

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- tests of general relativity
  - solar system tests, cosmological signatures, gravitational waves, . . .
- compatibility with standard model of particle physics at 1 TeV

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Structural demands

- resolution of spacetime singularities
- unitarity

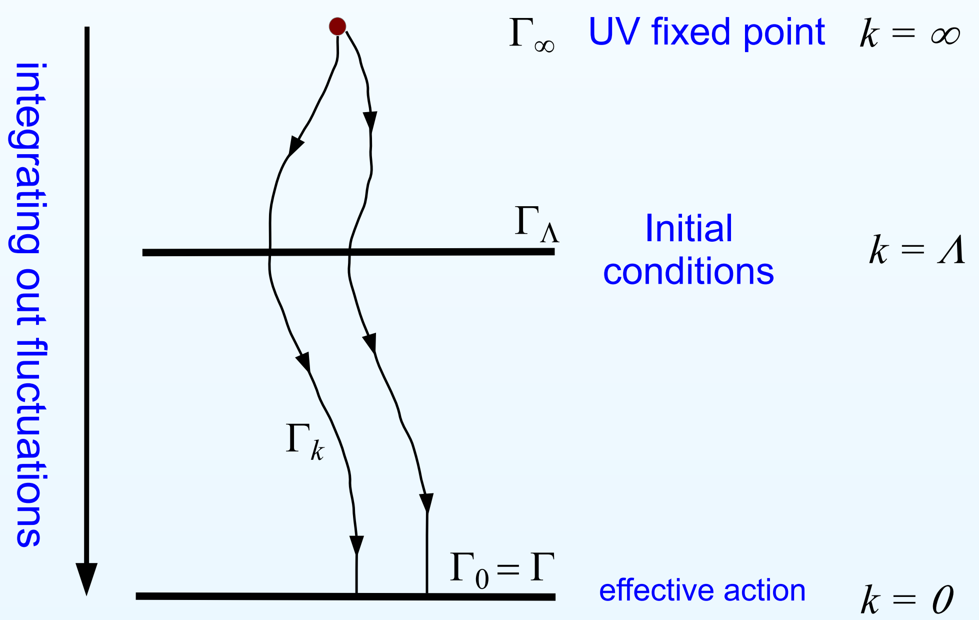
Exploring Asymptotic Safety via Functional Renormalization

C. Wetterich, Phys. Lett. **B301** (1993) 90
T. Morris, Int. J. Mod. Phys. A9 (1994) 24110
M. Reuter, Phys. Rev. D **57** (1998) 971

idea: integrate out quantum fluctuations shell-by-shell in momentum-space

implementation: Wetterich equation for effective average action Γ_k:

$$k\partial_k \Gamma_k[h_{\mu\nu}, \bar{g}_{\mu\nu}] = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$



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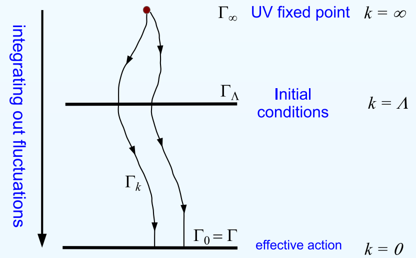
- realizes Wilson's idea of renormalization
- does not require specifying a "fundamental action"
  - fixed points come as solutions of the equation
- the effective action Γ is recovered for $k = 0$



Constructing approximate solutions of the flow equation

Ansatz for Γ_k restricting to a subset of all monomials

$$\Gamma_k[h_{\mu\nu}; \bar{g}_{\mu\nu}] = \sum_{i=1}^{N} \bar{u}_i(k)\, \mathcal{O}_i[h_{\mu\nu}; \bar{g}_{\mu\nu}]$$

$\Longrightarrow$ projection of flow onto ansatz gives β-functions for $\bar{u}_i(k)$

$$k\partial_k \bar{u}_i(k) = \beta_i(\bar{u}_i; k)$$

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systematic expansion schemes (choosing a basis $\{\mathcal{O}_i\}$):

- geometry focused:

$$\Gamma_k[h_{\mu\nu};\bar{g}_{\mu\nu}] = \frac{1}{16\pi G_k}\int d^4x\sqrt{g}\,[2\Lambda_k - R] + \cdots$$

- focus on correlation functions of the fluctuation fields:

$$\Gamma_k[h_{\mu\nu};\bar{g}_{\mu\nu}] = \sum_{n=1}(\prod_{i=1}^{n}\int d^4x_i\sqrt{\bar{g}})\,\Gamma_k^{(n)}[\bar{g}_{\mu\nu}]\,h_{\mu_1\nu_1}\times\cdots\times h_{\mu_n\nu_n}$$

Asymptotic Safety: contemporary misconceptions

[B. Knorr, C. Ripken, F.S., arXiv:1907.02903]
[J. F. Donoghue, arXiv:1911.02967]
[T. Draper, B. Knorr, C. Ripken, F.S., in preparation]

Is Newton's coupling momentum-dependent?

- physics comes from the effective action $\Gamma \equiv \Gamma_{k=0}$

$$\Gamma = \frac{1}{16\pi G} \int d^4x \sqrt{g} \left[2\Lambda - R + C_{\mu\nu\rho\sigma} W^C(\Delta) C^{\mu\nu\rho\sigma} - R W^R(\Delta) R \right] + \mathcal{O}(R^3)$$

- the "renormalized" G and Λ are numbers without momentum dependence
- momentum-dependent corrections are provided by form factors

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- the "renormalized" G and Λ are numbers without momentum dependence
- momentum-dependent corrections are provided by form factors

Constructing $G(E)$ from different scattering amplitudes leads to inconsistencies:

- The energy-dependence must be attributed to form factors
- Different amplitudes depend on different combinations of form factors
  $\Longrightarrow$ no inconsistency

Open Questions

- number of free parameters?
- exact matter content supporting asymptotic safety?
- degrees of freedom associated with the NGFP?

- low-energy physics compatible with observations?
- unitarity?

- Asymptotic Safety with Lorentzian signature metrics?

- characterization of the quantum geometry associated with NGFP?

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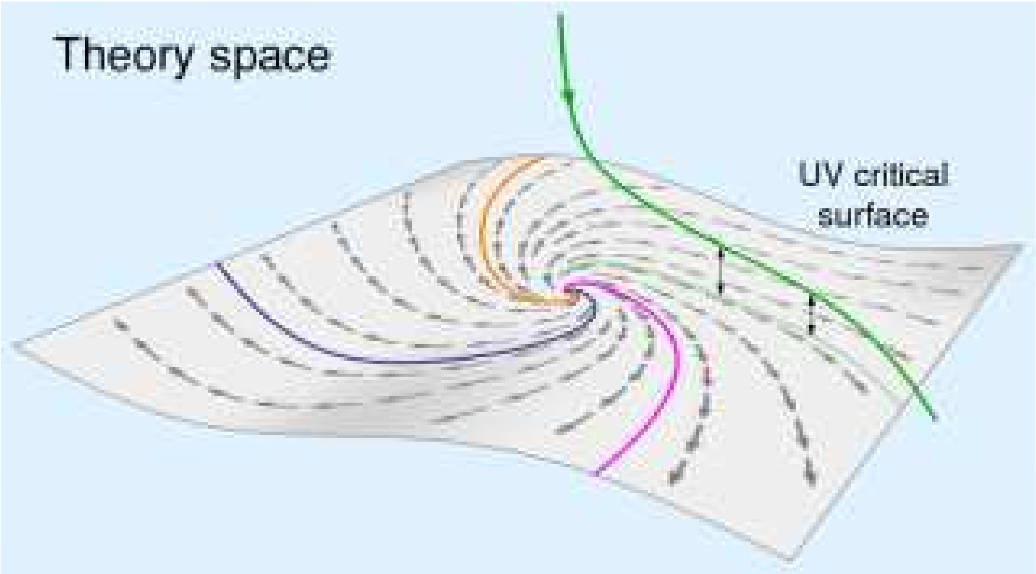
RG flow at the Reuter fixed point

Relevant parameters of a NGFP



strategy:

- linearize flow at the fixed point
- study spectrum of the stability matrix
  - eigenvalues with Re$(\lambda) < 0$: free parameter
  - eigenvalues with Re$(\lambda) > 0$: prediction for a coupling

Relevant parameters of a NGFP: practical search

project Wetterich equation on ansatz for Γ_k containing couplings $\{u_n\}$

$\Downarrow$ obtain beta functions

$$k\partial_k u_n = \beta_{u_n}(\{u_m\})$$

$\Downarrow$ identify fixed points

$$\beta_{u_n}(\{u_m\})\Big|_{u=u^*} = 0\,, \qquad \forall n$$

$\Downarrow$ determine eigenvalues $\{\lambda_n\}$ of the stability matrix

$$B_{nm} \equiv \frac{\partial \beta_{u_m}}{\partial u_n}\Big|_{u=u^*}$$

- spectrum is invariant under redefining coordinates

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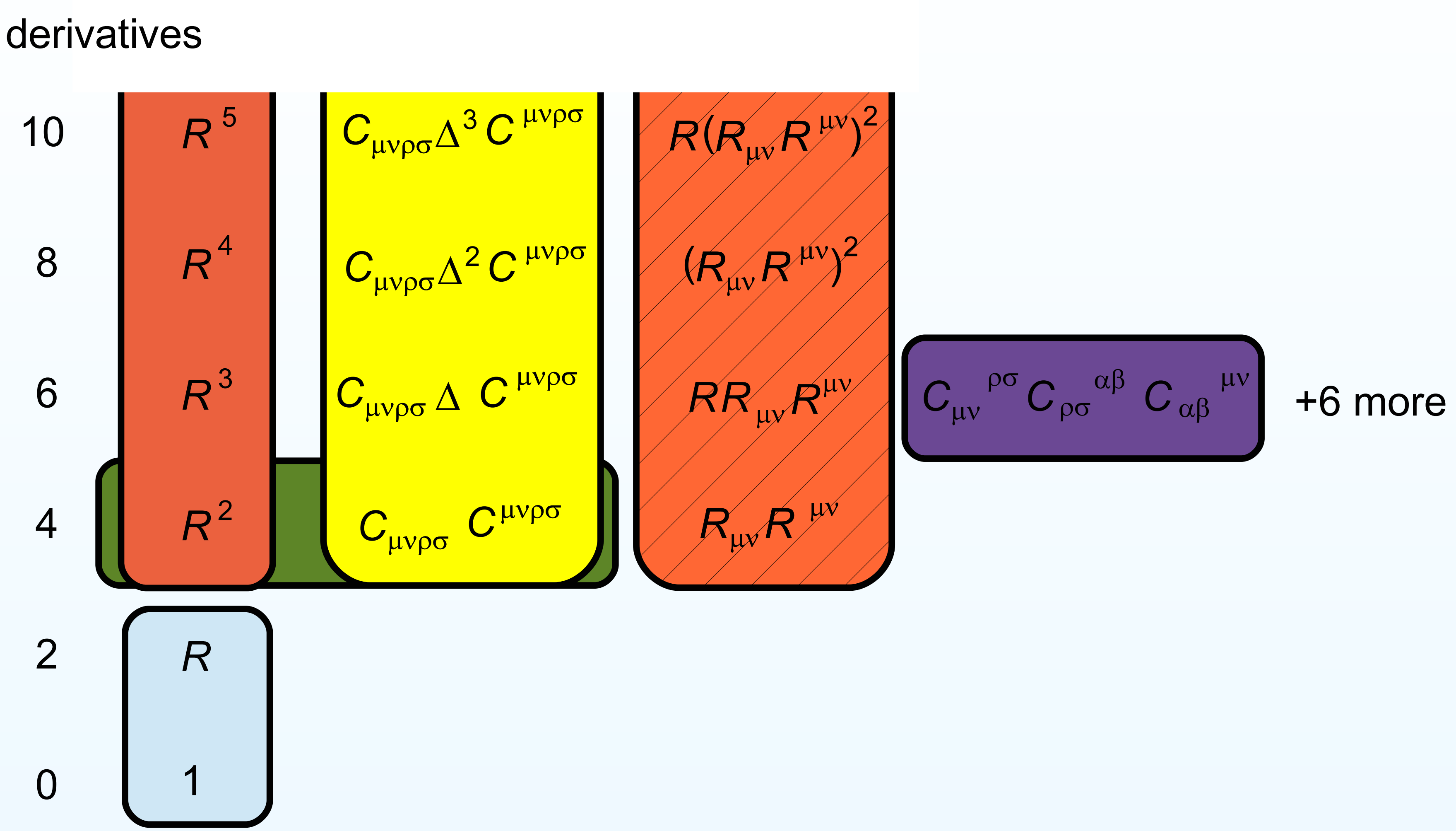
complications:

- computations get technically involved quickly
- spurious fixed point solutions

derivative expansion of Γ_k: FKWC-basis



derivative expansion of Γ_k: FKWC-basis

derivatives					
10	R^5	$C_{\mu\nu\rho\sigma}\Delta^3 C^{\mu\nu\rho\sigma}$	$R(R_{\mu\nu}R^{\mu\nu})^2$		
8	R^4	$C_{\mu\nu\rho\sigma}\Delta^2 C^{\mu\nu\rho\sigma}$	$(R_{\mu\nu}R^{\mu\nu})^2$		
6	R^3	$C_{\mu\nu\rho\sigma}\Delta\, C^{\mu\nu\rho\sigma}$	$RR_{\mu\nu}R^{\mu\nu}$	$C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\alpha\beta} C_{\alpha\beta}{}^{\mu\nu}$	+6 more
4	R^2	$C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma}$	$R_{\mu\nu}R^{\mu\nu}$		
2	R				
0	1				

today: limit to spherically symmetric backgrounds

Polynomial expansion of $f(R)$-gravity

[A. Codello, R. Percacci, C. Rahmede,arXiv:0705.1769]
[P. Machado, F. Saueressig, arXiv:0712.0445]
[K. Falls, D. Litim, J. Schröder, arXiv:1810.08550]

$$\Gamma_k[g,\bar{g}] = \int d^4x\,\sqrt{g}\,\sum_{n=0}^{N} u_n\,(R/k^2)^n\,k^4 + \cdots$$

- linearized RG flow at NGFP $\Longrightarrow$ three UV relevant directions

N	Re $\lambda_{0,1}$	Im $\lambda_{0,1}$	λ_2	λ_3	λ_4	λ_5
1	-2.38	2.17				
2	-1.26	2.44	-27.0			
3	-2.67	2.26	-2.07	4.42		
4	-2.83	2.42	-1.54	4.28	5.09	
5	-2.57	2.67	-1.73	4.40	3.97 + $4.57i$	3.97 - $4.57i$

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$N = 71$: for large n, eigenvalues exhibit an “almost Gaussian scaling”

$$\mathrm{Re}(\lambda_n) \approx a n - b \,, \qquad a = 2.042 \,, \quad b = 2.91$$

The composite operator formalism

[J. Pawlowski, hep-th/0512261]

[C. Pagani, M. Reuter, arXiv:1611.06522]

[C. Pagani, H. Sonoda, arXiv:1707.09138]

[M. Becker, C. Pagani, O. Zanusso, arXiv:1911.02415]

⋮

composite operator equation

idea: study properties of operators typically not contained in Γ_k

$$\langle \mathcal{O} \rangle = N \int \mathcal{D}\chi \, \mathcal{O} \, e^{-S[\chi]} = -\frac{\delta}{\delta \epsilon} N \int \mathcal{D}\chi e^{-S[\chi] - \epsilon \cdot \mathcal{O}}$$

for the effective average action

$$k\partial_k \left(\frac{\delta}{\delta \epsilon} \Gamma_k[\epsilon] \right)_{\epsilon=0} = -\frac{1}{2} \mathrm{Tr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \frac{\delta \Gamma_k^{(2)}}{\delta \epsilon} \left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} k\partial_k \mathcal{R}_k \right]_{\epsilon=0}$$

- scale-dependence of $\mathcal{O}$ determined by a single operator insertion

generalization to sets of operators $\mathcal{O}_n(k) \equiv \sum_m^N Z_{nm}(k) \, \mathcal{O}_m$:

$$\sum_{m=1}^{N} \gamma_{nm} \, \mathcal{O}_m = -\frac{1}{2} \mathrm{Tr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \mathcal{O}_n^{(2)} \left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \partial_t \mathcal{R}_k \right] ,$$

- $\gamma_{nm} \equiv \left(Z^{-1} \partial_t Z \right)_{nm}$ are generalized anomalous dimensions

Relation between γ_{nm} and the stability matrix

- consider the Wetterich equation for $\Gamma_k = \bar{\Gamma}_k(u_i) + \epsilon\, \bar{u}\, \mathcal{O}$
- expansion to first order in ϵ:

$$k\partial_k \bar{u}\, \mathcal{O} = -\frac{1}{2}\mathrm{Tr}\left[(\bar{\Gamma}_k^{(2)} + \mathcal{R}_k)^{-1}\, \bar{u}\, \mathcal{O}^{(2)} (\bar{\Gamma}_k^{(2)} + \mathcal{R}_k)^{-1}\, k\partial_k \mathcal{R}_k\right]\Big|_{\mathcal{O}}$$

- convert to dimensionless couplings $u = \bar{u} k^{-d_u}$:

$$(k\partial_k u + d_u\, u)\, \mathcal{O} = -\frac{1}{2}\mathrm{Tr}\left[(\bar{\Gamma}_k^{(2)} + \mathcal{R}_k)^{-1}\, u\, \mathcal{O}^{(2)} (\bar{\Gamma}_k^{(2)} + \mathcal{R}_k)^{-1}\, k\partial_k \mathcal{R}_k\right]\Big|_{\mathcal{O}}$$

- using the definition of γ_{uu}:

$$k\partial_k u = (\gamma_{uu} - d_u)\; u = \beta_u(u_i)$$

- obtain the stability matrix by taking a derivative with respect to u:

$$B_{uu} = \underbrace{-d_u}_{\text{classical}} + \underbrace{\gamma_{uu}(u_i)}_{\text{quantum}}$$

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generalization to a set of operators $\{\mathcal{O}_n\}$

$$B_{nm} = -d_n \delta_{nm} + \gamma_{nm}$$

The composite operator formalism

actual computation

input for the composite operator equation

1) choice of operators $\mathcal{O}_n$:

$$\mathcal{O}_n = \int d^{\rm d}x \sqrt{g}\, R^n \,, \qquad\qquad n \geq 0 \in \mathbb{N}$$

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- gauge-fixed Einstein-Hilbert action

3) optional: position of the Reuter fixed point (Einstein-Hilbert truncation)

- harmonic gauge:

$$d = 3: \qquad g^* = 0.199\,, \qquad \lambda^* = 0.063$$

$$d = 4: \qquad g^* = 0.707\,, \qquad \lambda^* = 0.193$$

- geometric Landau gauge:

$$d = 3: \qquad g^* = 0.198\,, \qquad \lambda^* = 0.042$$

$$d = 4: \qquad g^* = 0.911\,, \qquad \lambda^* = 0.160$$

anomalous dimension matrix: general structure

$\mathcal{O}_n$ contains at least R^{n-2} powers of the scalar curvature

$\Longrightarrow$ γ_{nm} has a triangular structure

$$\gamma(g,\lambda;k) = \begin{bmatrix} \gamma_{00} & \gamma_{01} & \gamma_{02} & \gamma_{03} & \gamma_{04} & \gamma_{05} & \gamma_{06} & \cdots \\ \gamma_{10} & \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} & \gamma_{15} & \gamma_{16} & \cdots \\ \gamma_{20} & \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} & \gamma_{25} & \gamma_{26} & \cdots \\ 0 & \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} & \gamma_{35} & \gamma_{36} & \cdots \\ 0 & 0 & \gamma_{42} & \gamma_{43} & \gamma_{44} & \gamma_{45} & \gamma_{46} & \cdots \\ 0 & 0 & 0 & \gamma_{53} & \gamma_{54} & \gamma_{55} & \gamma_{56} & \cdots \end{bmatrix} .$$

- γ is independent of the position of the Reuter fixed point

  $\Longrightarrow$ assigns a set of anomalous dimensions to any point in the g-λ-plane

- $\mathcal{O}_n$ are not eigenvectors of γ

- γ turns sparse as $N \to \infty$

Results I: single-operator approximation

select $\mathcal{O}_n$ with n fixed $\Longleftrightarrow$ compute diagonal entries γ_{nn}:

$$\gamma_{nn} = \frac{32\pi g}{(4\pi)^{d/2}} \left[A_0(g,\lambda;d) + A_1(g,\lambda;d)\, n + A_2(g,\lambda;d)\, n^2 \right]$$

- second-order polynomial in n
- coefficients A_i are known analytically
- strength of anomalous dimension controlled by g

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explicit expressions at the Reuter fixed point:

- harmonic gauge:

$$d = 3: \qquad \gamma^*_{nn} = 1.59 - 1.51n - 0.12n^2$$

$$d = 4: \qquad \gamma^*_{nn} = 3.99 - 4.73n - 0.10n^2$$

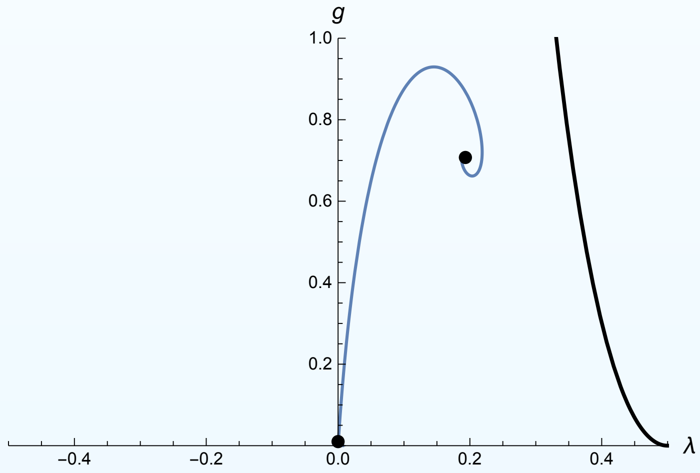
- geometric Landau gauge:

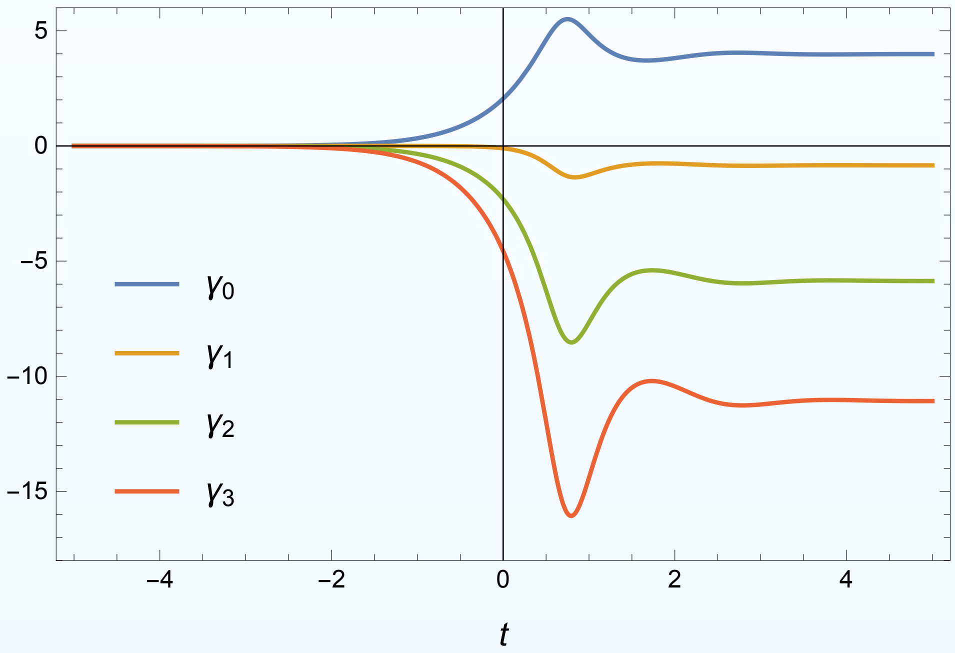
$$d = 3: \qquad \gamma^*_{nn} = 0.65 - 0.87n - 0.03n^2$$

$$d = 4: \qquad \gamma^*_{nn} = 2.30 - 3.77n$$

Results II: full matrix γ_{mn}

evaluate γ_{nn} along a RG trajectory of Type IIa:

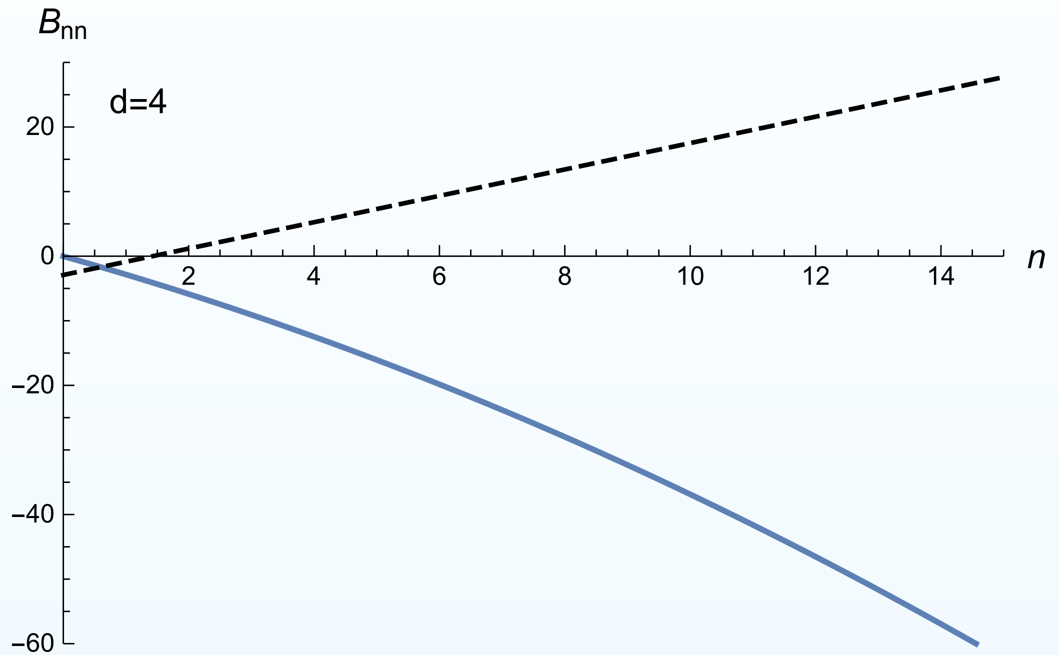




obtain classical geometry at the GFP

Results I: single-operator approximation

diagonal entries of the stability matrix: $B_{nn} = -d_n + \gamma^*_{nn}$



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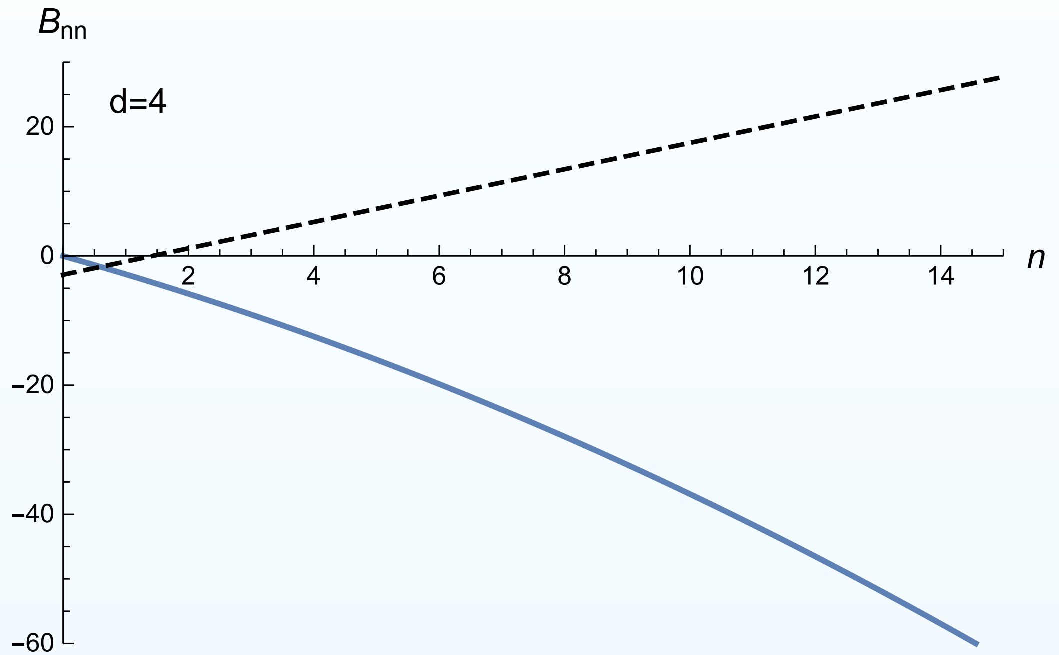

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B_{nn} are not the eigenvalues of $B \Longleftrightarrow$ off-diagonal terms matter!

The composite operator formalism

the full matrix γ

Results II: the generating functional for γ_{nm}

composite operator equation on a d-sphere background:

$$\sum_m \gamma_{nm} \mathcal{O}_m = \int d^d x \sqrt{g} \quad \underbrace{\Gamma_n(\bar{R})}_{\text{operator traces of Laplacians}}$$

- adapt geometric Landau gauge
- truncated heat-kernel at $O(\bar{R}^2)$

generating functionals can be given explicitly

$$\Gamma_n(\bar{R}) = \frac{16\pi g}{(4\pi)^{d/2}} \left[\sum \text{threshold functions} \right]$$

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$$\sum_m \gamma_{nm} \mathcal{O}_m = \int d^d x \sqrt{g} \quad \underbrace{\Gamma_n(\bar{R})}_{\text{operator traces of Laplacians}}$$

- adapt geometric Landau gauge
- truncated heat-kernel at $O(\bar{R}^2)$

generating functionals can be given explicitly

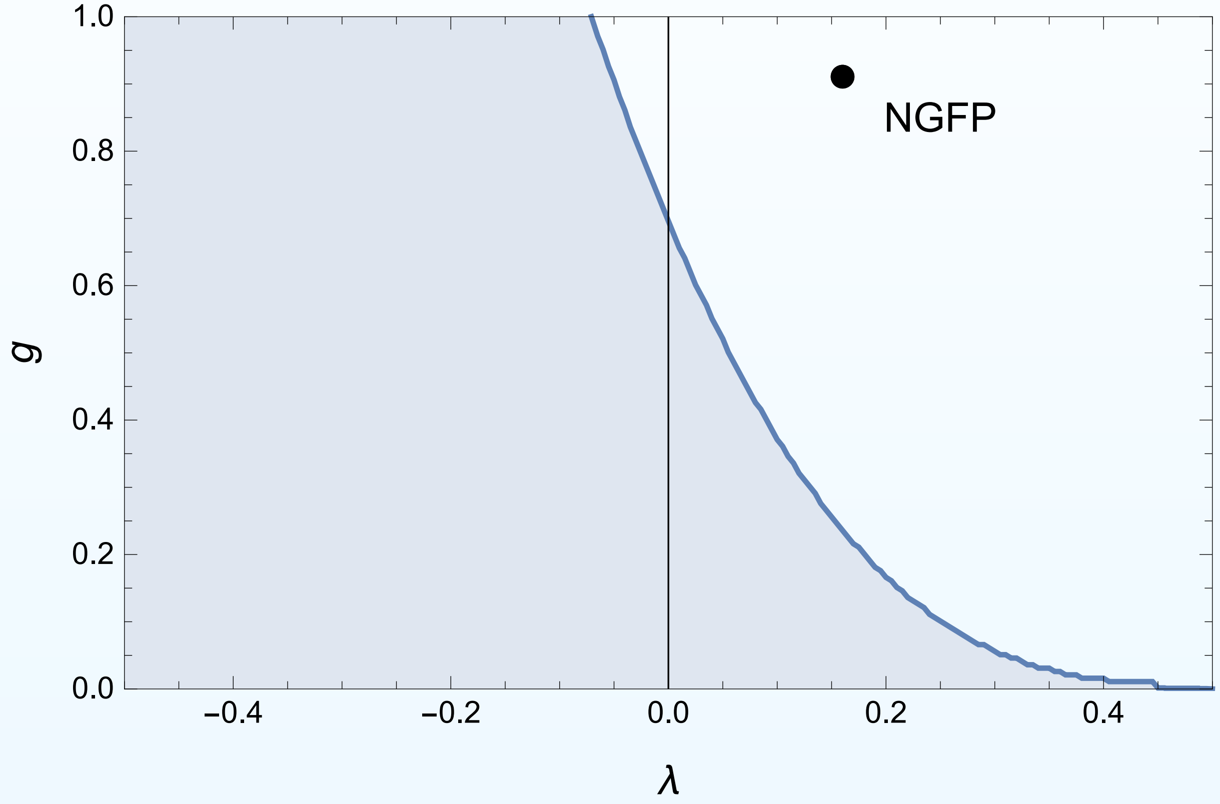
$$\Gamma_n(\bar{R}) = \frac{16\pi g}{(4\pi)^{d/2}} \Big[\sum \text{threshold functions} \Big]$$

generating functionals $\Gamma_n(\bar{R})$ encode B_{nm}

study spec(B) for "large truncations" $N \simeq 100$

Results II: the perturbative region in the g-λ-plane

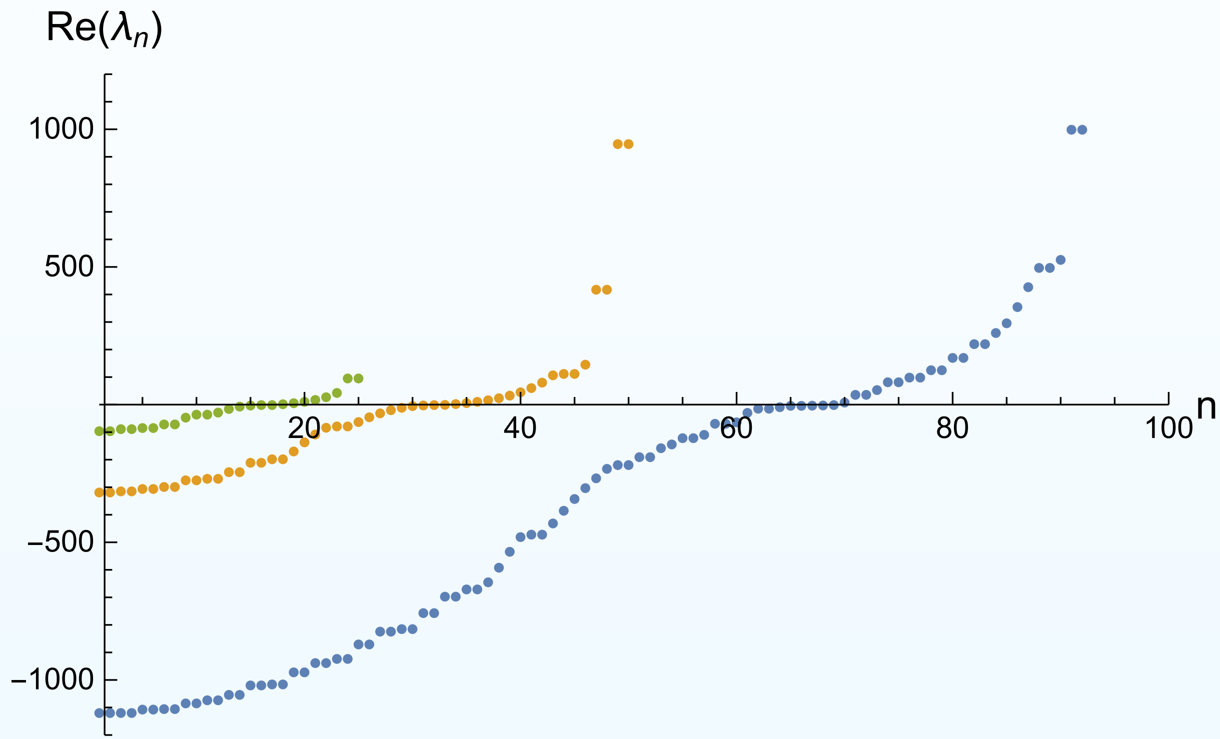
consider spec(B) for $N = 10$ and $d = 4$:



- shaded region: 3 relevant eigenvalues

Results II: tracing eigenvalues at the Reuter fixed point

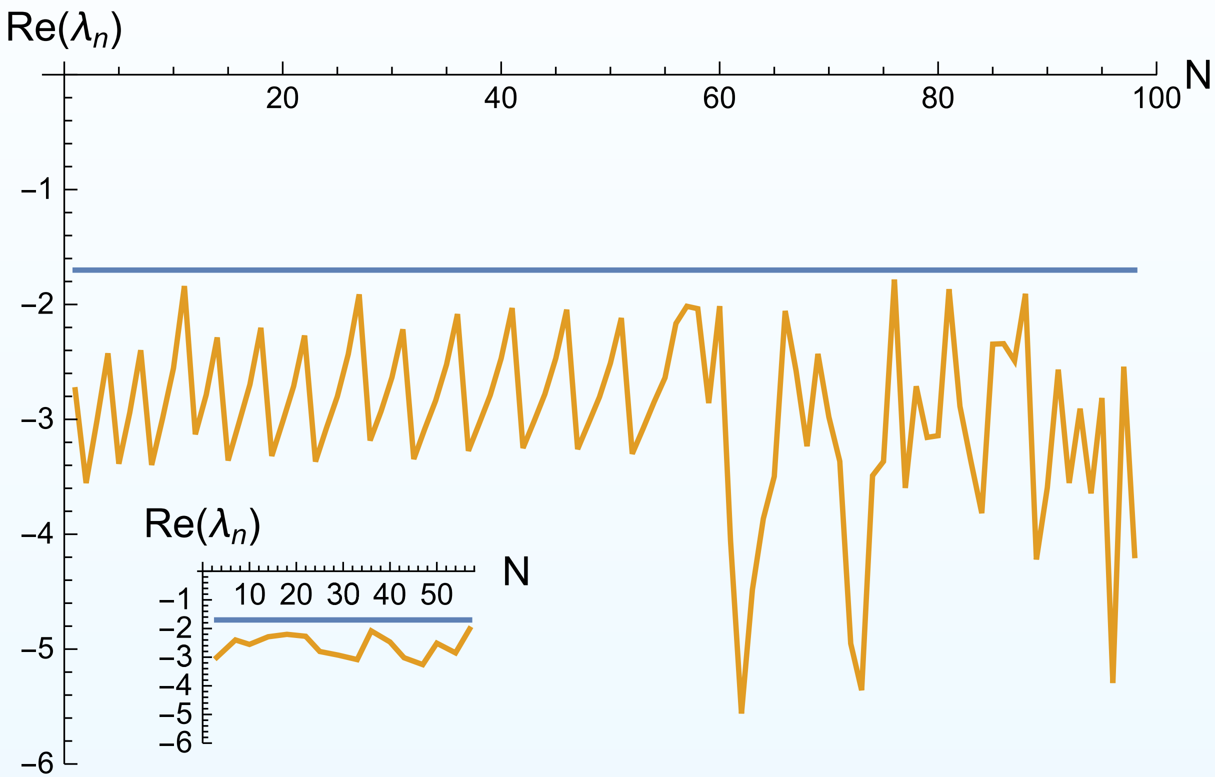
spectrum of B for $N = 25$, $N = 50$, $N = 100$



- the stability matrix is not bounded from below

Results II: tracing eigenvalues at the Reuter fixed point

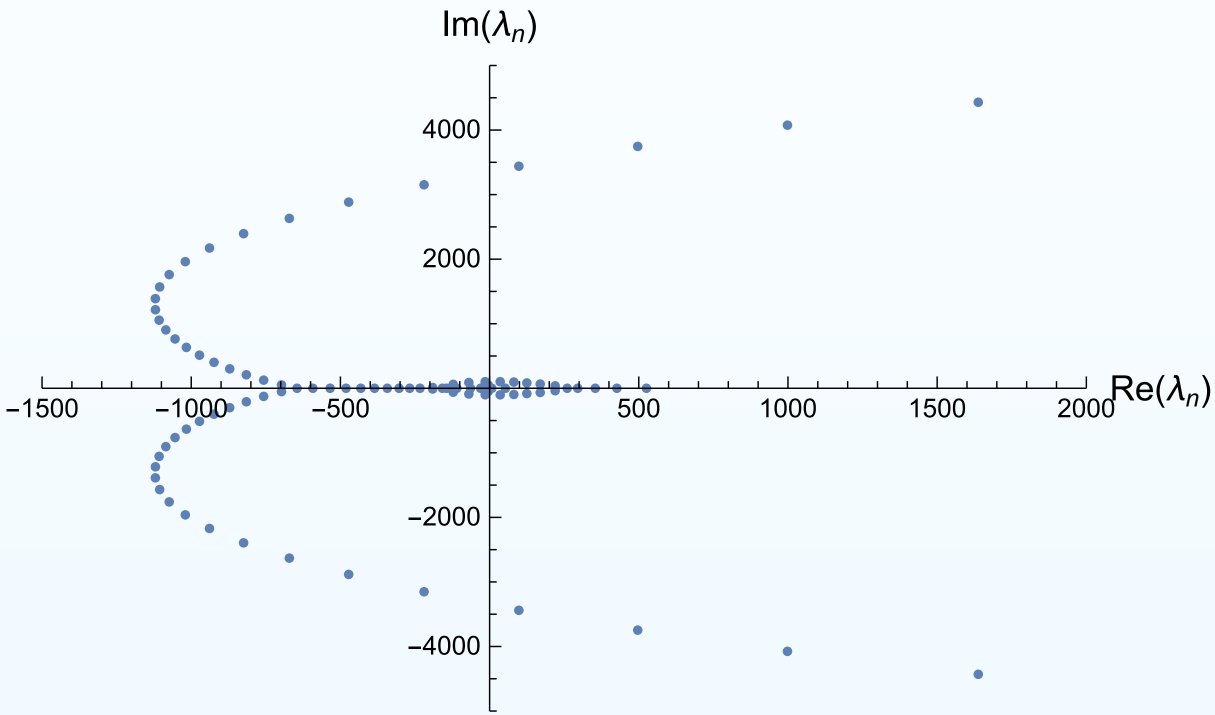
tracing the eigenvalues of $N = 2$ with the matrix size



- no obvious convergence pattern?

Results II: tracing eigenvalues at the Reuter fixed point

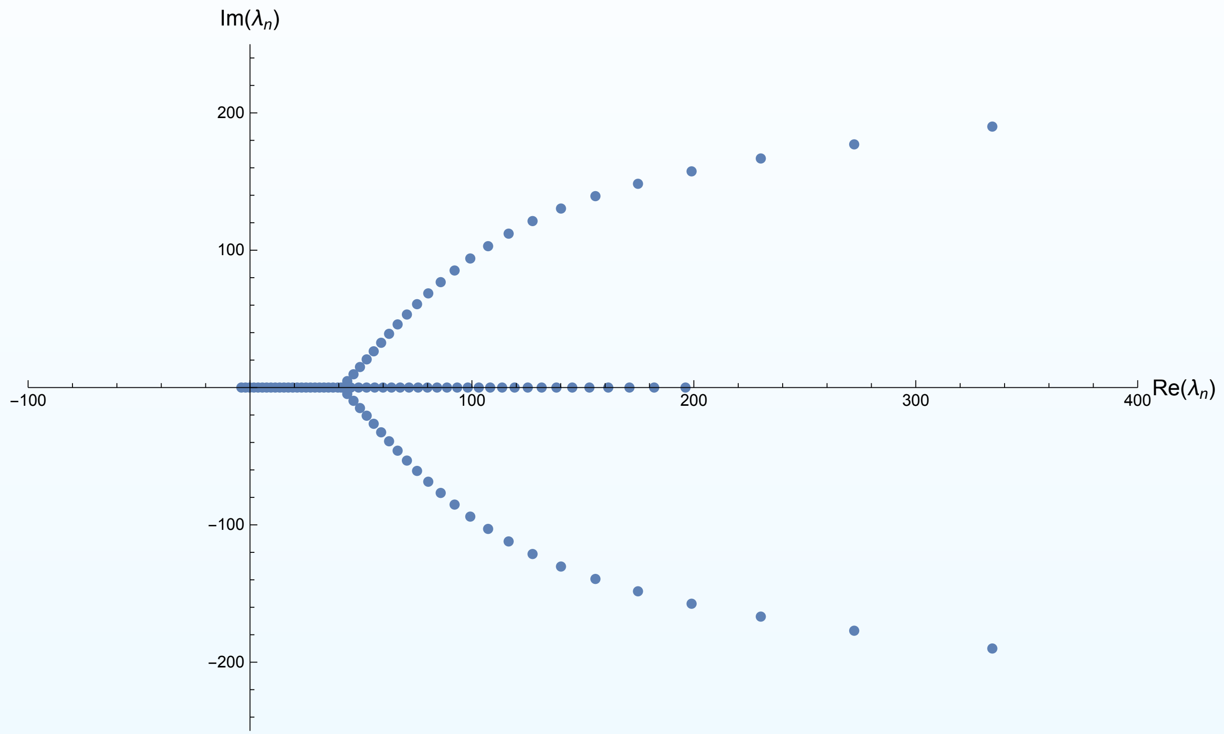
spectrum of B for $N = 100$



- eigenvalues follow distinct curves in the complex plane

Results II: tracing eigenvalues at the Reuter fixed point

spectrum of B for $N = 100$ at $g = 0.1, \lambda = 0$ (perturbative region)



- eigenvalue pattern depends on g, λ
- adding matter allows to shift fixed point position without changing γ_{nm}
  - identify characteristic eigenvalue patterns

Concluding Thoughts

connecting to random geometry

problem: scale k is difficult to access

$\Longrightarrow$ use relative scaling of operators

- suppose in the scaling regime $\mathcal{O}_n \propto k^{d_n+\gamma_n}$
- set scale via a reference operator, i.e., $\mathcal{O}_0$
- express scaling relative to this reference operator

$$\mathcal{O}_n \propto (\mathcal{O}_0)^{\frac{d_n+\gamma_n}{d_0+\gamma_0}} \qquad \Longleftrightarrow \qquad \tilde{\gamma}_n \equiv \frac{d_n+\gamma_n}{d_0+\gamma_0}$$

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predictions from the single-operator approximation in $d = 4$:

GFP: $\tilde{\gamma}_1 = 0.5$ NGFP: $\tilde{\gamma}_1 = 0.145$

summary: composite operator formalism

encouraging results $\cdots$

- complementary way to analyze stability properties of NGFPs
- observation: novel convergence patterns in spec(B)
- partial agreement with polynomial $f(R)$-results

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$\cdots$ and plenty to do:

- improve propagators
- consider other composite operators (geodesic distance,$\cdots$)

  [C. Pagani, M. Reuter, arXiv:1611.06522]

- other universality classes (Stelle gravity,$\cdots$)

  [M. Becker, C. Pagani, O. Zanusso, arXiv:1911.06522]

- classify gravity-matter fixed points according to eigenvalue patterns
- measurements in random geometry (2-dimensions, $2+1$-dimensions, $\cdots$)

Thank you