An introduction to covariant quantum gravity and asymptotic safety

List of changes

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Updated version of this file can be found in
`http://www.percacci.it/roberto/physics/book2/index.html`
Please send further corrections to `percacci@sissa.it`

• p.14, line below eq. (2.1): $G = 6.674 \times 10^{-11}$

• p.15, equation (2.9): some of the indices are not in the correct positions. The corrected formula reads

$$S_{FP} = \int d^4x \left(-\frac{1}{2}\partial_\alpha \phi_{\mu\nu} \partial^\alpha \phi^{\mu\nu} + \partial_\alpha \phi_\mu{}^\alpha \partial_\beta \phi^{\mu\beta} - \partial_\alpha \phi_\mu{}^\alpha \partial^\mu \phi + \frac{1}{2}\partial_\alpha \phi \partial^\alpha \phi \right) . \qquad (2.9)$$

• p.21, 8th line of second paragraph: replace "working on the physical principles" by "working on the same physical principles"

• p21, before and after eq. (2.46), and p.23, second line of second paragraph: replace "irrreducible representations of te Lorentz group" by "irreducible representations of the rotation group".

• p23, before equation (2.61) replace "wave-number" by "wave-vector"

• p24, third line: replace "tranformations" by "transformations"

• p24, before (2.62) modify te text into "distinguish generic gauge transformations..., from those for which it is also transverse."

• p25, eq. (2.69): replace $\partial^\nu \phi$ by $\partial^\mu \phi$

• p27, three lines below (2.84): change sign in the formula in line, to $2R_{\mu\nu}R^{\mu\nu} - \frac{2}{3}R^2$, and change $a_1 = 3a_2$ to $a_1 = -3a_2$.

• p.34, eq. (2.106): the font of 2 is corrected by changing the overall definition of `\lc` to `\newcommand{\lc}{{\mit\Gamma}}`

• p.34, unnumbered formula before (2.107): insert a factor 4! in the r.h.s.

• p.36, eqs.(3.2) and (3.9): replace d^4x by d^dx

• p.37, unnumbered formula before (3.11): N to be replaced by $\mathcal{N}$.

• p.39, equation (3.22): before the second equality, insert $|_{s=0}$.

• p.42, before eq. (3.37) replace (15.2.1) by (3.21) and (15.2.6) by (3.31).

• p.45, eq. (3.49): insert λ in the first term in square brackets.

• p.47, in the line after equation (3.60): replace "fuction" by "function".

• p.50, in the third last line: "the integrals of C^2 and E are Weyl invariant".

• p.51: in eq. (3.82), the classical action has to be added in the first two lines:

$$\begin{aligned} \Gamma(\Omega^2 g) &= S(\Omega^2 g) + \frac{1}{2}\sum_n \log\left(\frac{\Omega^{-2}\lambda_n}{\mu^2}\right) \\ &= S(g) + \frac{1}{2}\sum_n \log\left(\frac{\lambda_n}{\mu^2}\right) + \frac{1}{2}\log(\Omega^{-2})\mathrm{Tr}\mathbf{1} \\ &= \Gamma(g) - \log\Omega\,\zeta_\Delta(0) \ , \end{aligned}$$

• p.53: replace the text after (3.100) and equation (3.101) by:

We can now interchange the f and h integrations and use the finite analogues of (3.94-98-99) to change h to h^f in the measure, in Ψ and in the action:

$$Z = \int (df) \int_{\mathcal{M}} (dh^f) \Psi(h^f; \bar{g}) \delta(F_\mu(h^f; \bar{g})) e^{iS(h^f; \bar{g})} \ . \tag{1}$$

• p.55: in eq. (3.110) and (3.111), in the exponents on the l.h.s. one has $W(j, \bar{\tau}, \tau; \bar{g})$.

• p.57: in eq. (3.124) there is an undefined quantity h_μ. Also $\Box$ should be typeset $\bar{\nabla}^2$. The corrected formula reads as follows:

$$R^{(2)} \approx \frac{1}{4}(h_{\mu\nu}\bar{\nabla}^2 h^{\mu\nu} + h\bar{\nabla}^2 h + 2\bar{\nabla}_\rho h^\rho{}_\mu \bar{\nabla}_\sigma h^{\sigma\mu} + 2\bar{R}_{\alpha\beta} h^{\alpha\gamma} h^\beta_\gamma + 2\bar{R}_{\alpha\beta\gamma\delta} h^{\alpha\gamma} h^{\beta\delta}). \tag{3.124}$$

• p.62, eq.(3.158): replace 10 by 20, twice.

• p.67, eq.(3.174): replace $-16\frac{4}{t^2}$ by $+\frac{4}{t^2}$. In the next unnumbered formula, change the sign of the second term on the r.h.s.

• p.80, fig.4.1: remove the first two figures on the left. Three-point vertices are present for more general parametrizations of the target space, but are absent with the exponential parametrization (4.6). In the subsequent line remove "two, three or four".

• p.86, four lines below eq. (4.20): replace "which is subject" by "which are subject".

• p.87, in eq. (4.22): replace $\sqrt{g}$ by $\sqrt{|g|}$.

• p.99, eq. (5.60): the initial factor 1/2 multiplies all that follows:

$$R^{(1)}_{\mu\nu} = \frac{1}{2}\left[\Delta_{L2} h_{\mu\nu} + \nabla_\mu \left(\nabla_\rho h^\rho{}_\nu - \frac{1}{2}\nabla_\nu h\right) + \nabla_\nu \left(\nabla_\rho h^\rho{}_\mu - \frac{1}{2}\nabla_\mu h\right)\right] \ . \tag{5.60}$$

• p.100, equations (5.63-64) hold in an Einstein space. Equations (5.61-62-65) actually hold for any metric.

• p.112, in eq.(5.113) replace $-\frac{2\bar{R}}{d}$ by $-\frac{\bar{R}}{2}$.

• p.112, two lines before eq.(5.114) replace "are witten" by "are written".

• p.115, 2nd line of section 5.5.2, replace "Lichnewrowicz" by "Lichnerowicz".

• p.124, eq.(5.166): replace f by W.

• p.118, Table 5.1: replace everywhere l by ℓ.

• p.129, in the fifth last line of the introductory section, change the text to "...derivation of te Exact Renormalization Group Equation (ERGE)".

• p.141, eq.(6.50): add $-S_{gh}$ in the exponent.

• p.155, in the unnumbered matrix before eq.(6.106): replace $\frac{\delta^2\Gamma_k}{\delta a \delta a}$ by $\frac{\delta^2\Gamma_k}{\delta h \delta h}$.

• p.158, eq.(6.119): replace dx by $d^d x$.

• p.188, before eq.(7.50) remove the words "They do not change under Euclidean continuation" and after eq.(7.50) add: "(this is not the Wick rotation of equation (2.75), since the signs of α, β, γ would have to be changed)".

• p.193, equations (7.77-79): in the l.h.s. of each equation, remove the subscript h from Δ_h.

• p.195, after eq.(7.94): replace "which" by "that".

• p.198, in the first line of the second paragraph, remove the word "how"

• p.216, in the first line after table (7.1) the reference should be to eq. (7.146) rather than (6.116).

• p.218-221, in several formulas in section 7.6.3 the dependence on the parameters M^2 and Λ does not reflect the calculations described in the text.

– In equations (7.154) and (7.164-166), $\tilde{\Lambda}$ has to be replaced everywhere by $-\tilde{M}^2/2$.

– In the line after eq. (7.155), the text should be modified into: "The coefficients A_1, A_2, B_1, B_2 are still given by (6.121), except for the replacement of the denominators $1-2\tilde{\Lambda}$ by $1+\tilde{M}^2$"

– The functions a, b, c, d, e, f defined in equations (7.159) and (7.163) generally depend on $\tilde{\Lambda}$ and $\tilde{M}^2$.

– Equations (7.160) and (7.161) have to be replaced by the following:

$$
\begin{aligned}
a(\tilde{\Lambda},\tilde{M}^2) &= \frac{a_0+a_1\tilde{\Lambda}+a_2\tilde{\Lambda}^2+a_3\tilde{M}^2+a_4\tilde{M}^4+a_5\tilde{M}^6+a_6\tilde{M}^8+a_7\tilde{\Lambda}\tilde{M}^2}{(4\pi)^{d/2}\Gamma(d/2)d^2(d^2-4)(3d-2)(1+\tilde{M}^2)^4}\ , \\
a_0 &= -4\pi(d-2)(-896+264d+1076d^2-434d^3+21d^4+d^5)\ , \\
a_1 &= 16\pi(d-1)(d^2-4)(64+52d-32d^2+3d^3)\ , \\
a_2 &= -16\pi d(d+2)(-16+12d+40d^2-19d^3+d^4)\ , \\
a_3 &= -8\pi(d-2)(-2176+2512d-138d^2-111d^3-27d^4+4d^5)\ , \\
a_4 &= -32\pi(d-2)(-816+1180d-384d^2+55d^3-12d^4+d^5)\ ; \\
a_5 &= 4a_6=-512\pi(d-2)(-32+50d-19d^2+2d^3)\ ; \\
a_7 &= -64\pi(d-1)(d^2-4)(16+26d-13d^2+d^3)\ ; \\
c(\tilde{\Lambda},\tilde{M}^2) &= \frac{c_0+c_1\tilde{\Lambda}+c_2\tilde{M}^2}{(4\pi)^{d/2}\Gamma(d/2)\,d^2(d+2)(d+4)(3d-2)(1+\tilde{M}^2)^3} \\
c_0 &= 8\pi(d-1)(128+720d-350d^2+29d^3)\ ; \\
c_1 &= 64\pi(d-1)(d+4)(16+26d-13d^2+d^3)\ ; \\
c_2 &= 32\pi(d-1)(d-3)(d+4)(-8-10d+d^2)\ ;
\end{aligned}
$$

– In equation (7.167) add the term $+\eta_h\tilde{M}^2$

– Equation (7.170) has to be replaced by the following:

$$
r(\tilde{\Lambda},\tilde{M}^2)=\frac{512\pi d}{(4\pi)^{d/2}\Gamma(d/2)(d+4)(d+6)(3d-2)} \tag{2}
$$

These beta functions and anomalous dimensions give the flow described in fig.7.18 and Table 7.2. They reduce to the expressions given in the book when M^2 is identified with -2Λ.

• p.223, Table 7.2: the one loop value for $\tilde{M}_*^2$ is -0.335 instead of -0.565.

• p.228, Table 7.4: the correct title of the table is "Scaling exponents".

• p.253, the second last paragraph should begin with: "In an application to cosmology".

• p.282, ref.[224]: capitalize Gilkey.